Optimal Design of Multiperiod Batch-Storage Network Including Transportation Processes

Gyeongbeom Yi
Dept. of Chemical Engineering, Pukyong National University, Yongdang-Dong, Nam-Ku, Busan 608-739, Korea

Gintaras V. Reklaitis
School of Chemical Engineering, Purdue University, West Lafayette, IN 47907

DOI 10.1002/aic.12492
Published online February 2, 2011 in Wiley Online Library (wileyonlinelibrary.com).

An optimally designed of batch-storage network that uses a periodic square wave model provides analytical lot-sizing equations for a complex supply chain network characterized as a multisupplier, multiproduct, multi-stage, nonserial, multicustomer, cyclic system, with recycling or remanufacturing. The network structure includes multiple currency flows and material flows. The processes involve multiple feedstock and product materials with fixed compositions that are highly suitable for production processes. Transportation processes that carry multiple materials of unknown composition are included in this study, and the time frame is varied from a single infinite period to multiple finite periods to accommodate nonperiodic parameter variations. The objective function in the optimization is chosen to minimize the difference between the opportunity costs of currency/material inventories and stockholder benefits given in the numeraire currency. Expressions for the Kuhn-Tucker conditions for the optimization problem are reduced to a multiperiod subproblem describing the average flow rates and the analytical lot-sizing equations. The multiperiod lot-sizing equations are shown to differ from their single-period counterparts. The multiperiod subproblem yields a multiperiod planning model that has many advantages over existing planning models. For example, it contains terms that represent operation frequency dependent costs. Realistically sized numerical examples that deal with multinational corporations are formulated and tested. The effects of corporate income taxes, interest rates, and exchange rates are presented. © 2011 American Institute of Chemical Engineers *AIChE J,* 57: 2821–2840, 2011

Keywords: optimal lot-size, process-inventory network, multiperiod planning model, transportation processes, multinational corporation

Introduction

A supply chain structure can be approximated as a batch-storage network (BSN). BSN can account for most supply chain components, for example the purchase of raw materials, production, transportation, and demand for the finished product. The periodic square wave (PSW) model has been used successfully to find analytical solutions for a variety of supply chain systems, including a parallel system with batch processes and storage units,[1] a sequential multistage BSN,[2] a nonsequential BSN including recycling streams,[3] a BSN supported by financial transactions and cash flows,[4,5] multitasking semi-continuous processes in a BSN,[6] a BSN that considers uncertainty and waste streams,[7,8] a multisite BSN,[9] and a BSN that includes the effects of exchange rates and taxes on a multinational corporation (MNC).[5,10]

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Correspondence concerning this article should be addressed to G. Yi at gbyi@pknu.ac.kr.

This study contributes to the optimal design of BSNs in three ways. First, a basic assumption of the PSW model is that all operations are periodic. However, this assumption must be violated in the long term, because the annual demand rates for petrochemical products, for example, are generally increasing. A possible remedy is to introduce a multiperiod formulation into the PSW model. In this study, we introduce a multiperiod formulation into a PSW model that considers financial flows, and we demonstrate that the resulting lot-sizing equations differ from those derived using a single-period formulation. Second, the processes in the BSNs are designed to represent the production processes and the class of transportation processes that carry a single product. However, real world transportation processes can carry multiple products in multiple parcels at the same time, and it usually makes impossible to disaggregate input data by product. To cope with this situation, we introduce transportation processes that can carry multiple products in multiple parcels into the BSN structure. Third, we provide a strategic or tactical planning model based on the BSN and its special solution methods. The planning model based on the BSN has large numbers of binary variables and nonconvex nonseparable terms in the MINLP formulation. We use separable programming techniques to convert the MINLP into MILP that can be solvable with powerful commercial solvers.

Planning models have been used in the petrochemical industry since the 1950s and are often cited as a major triumph for applied mathematical programming.[11] Planning models are classified by timescale as strategic planning, tactical planning, or operations planning, and by application area as production planning or distribution planning. A production planning model helps the user to economically purchase raw materials and suitably adjust process operation levels in the near future, so that estimated finished product demand profiles are met within a plant. The constraints on the production planning model consist of material balance, inventory replenishment, and the capacity limits of equipment. Refinery plants have additional bilinear property constraints, which give rise to what is known as the pooling problem.[12]

The structure of a distribution planning model is essentially that of a transshipment problem.[13] The distribution planning model does not include the details of the processes within an individual plant. Instead, it treats each plant as a source of products, and plants located in different regions are treated as multiple sources of products. The products of source plants pass through several transshipment distribution centers and travel to a variety of destination terminals. Products are transported by several vehicle types, such as pipelines, ships, trains, and trucks. The constraints on the distribution model consist of the material balance within each facility, inventory replenishment equations and the throughput capacities of the facilities and vehicles. Binary variables are usually introduced to represent the opening and closing of facilities. The primary benefit of a distribution planning model is the reduction of transportation costs, which can be achieved by selecting the best routes and vehicles to meet regional product demands. Note that about 10 % of Shell's revenue is spent on transportation costs.[13] The distribution model also informs many strategic decisions, such as choosing the best facility locations, market territories, establishing reasonable vehicle capacities, and optimally negotiating product exchange.[14]

When planning models were applied to real world applications, such as those described above, computer hardware and MILP algorithms limited the practical size of models to a few thousand equations, defined over both a few thousand continuous variables and several hundred binary variables. By contrast, planning models in current use can involve as many as 370,500 equations, 400,666 continuous variables and 175,500 binary variables.[15] Note that the distinctions between classes of planning models and between planning and scheduling have been artificially determined based on the size of problems that could be dealt with at that times. Integrating the characteristics of several types of planning model into a single model is, therefore, an obvious way to enhance the benefits reaped through application of these models.[16,17] Current trends in the development of planning models can be summarized as: (1) integration of strategic, tactical, operations, production, and distribution planning models; (2) inclusion of important scheduling constraints, such as operation mode and timing; (3) usage of different time frames for high and low frequency operations in the multiperiod formulation; (4) inclusion of separable nonlinear terms that can be linearized, such as quantity dependent prices; and (5) dealing with demand and price uncertainties through stochastic programming.[18] During the last half-century, the size of optimization problems has increased significantly thanks to rapid advances in computer hardware and optimization algorithms. However, supply chain networks have also grown in physical size and complexity over the same period due to global economic growth, international trade, the merging of similar enterprises, and the globalization of enterprise business. For example, Dow Chemical Company has 156 manufacturing sites in 37 countries, produces more than 3300 products and sells the products to the customers in 175 countries.[19] As a result, the current computing capacity is insufficient for dealing with all relevant issues within a single model, and there is still a demand for efficient models that can cover most of the important issues.

The problem we are handling in this study is targeted to develop a multiperiod planning model for a BSN. Note that a BSN is well-suited to approximating a supply chain of MNC.[10] The basic elements of the BSN are production/transportation processes and material/currency storages. Therefore, BSN can represent the features of planning model in detail but is not enough to represent the features of scheduling model in the supply chain application. The supply chain system we are concerned in this study is composed of raw material suppliers, production plants, transportation vehicles, distribution centers, terminals, customers, and currency accounts spread all over many nations. We count on both material and financial flows with multiple currencies considering international financial factors such as currency exchange rates, corporate income taxes, and interest rates. The production plant is approximated by a complex network of production processes and material storages. The production processes consume multiple feedstock materials and produce multiple products. Materials are moved among production plants, distribution centers, and terminals by transportation processes across nation borders. The transportation processes have the capability of multiparcel and multiproduct delivery. Production plants, distribution centers, terminals, transportation processes, and currency account is owned by one of subsidiaries. Currencies are transferred among subsidiaries to pay for transferred material price. An MNC is composed of multiple subsidiaries spread all over many nations. The planning model presented here has many advantages over existing planning models described in the Refs. 11–19. For instance, it provides optimal operation frequencies without severe computational burden imposed by ordinary planning models, which require many binary variables to accomplish the same task. It can easily treat the costs that are dependent on operation frequency, such as setup costs and inventory holding costs, which also require many binary variables under ordinary planning models. Finally, through application of a method described previously,[8,9] the model can significantly reduce the computational burden required to analyze a variety of uncertainties. For the sake of brevity, discussion of some important issues, such as multitasking batch/semi-continuous processes and uncertainty, shall be included in a future study. We will restrict the type of production process to multiproduct batch processes. This discussion has been restricted to deterministic models. Additionally, we do not consider stockout costs.

The remainder of this article is organized as follows. First, the upper and lower bounds and the average batch material flow between a material storage unit and a (production or transportation) process are described using graphical methods. The upper and lower bounds and average batch currency flow between currency storage units are found by the same method. The upper and lower bounds and the average batch flow are used as constraints or terms, respectively, in the objective function defined in the subsequent optimization model. An optimization model that minimizes the difference between the opportunity costs of currency/material inventories and the benefit of stockholders in the numeraire currency is next introduced, and the analytical solutions to the problem, under the Kuhn-Tucker conditions for this optimization model, are found. The solutions are composed of analytical lot-sizing equations and a multiperiod planning model that has nonlinear, nonconvex, separable, and nonseparable terms in its constraints. The nonseparable terms in these constraints are replaced by adequate separable upper bound terms, and the final model is fully linearized by separable programming techniques. Finally, computational results that highlight the advantages of the proposed approach are presented, followed by the conclusions of this work.

Definitions of Variables and Parameters

This study applies a BSN to the internationally distributed production plants and logistic facilities of an MNC; we briefly define the variables and some of the equations used in this study below. T time periods are defined from $\tau = 0$ to $\tau = T - 1$, with time intervals $\bigtriangledown t^{[\tau]}$. It is not necessary for all $\bigtriangledown t^{[\tau]}$ to be equal with respect to τ. As depicted in Figure 1, a supply chain system that converts raw materials into final products through multiple physicochemical processing steps and subsequently transports them to customers is composed of a set of currency storage units (R, ellipse), a set of material storage units (J, circle), a set of batch processes (I, square), and a set of transportation processes (N, triangle). Semi-continuous processes may be included in the model with additional mathematical treatment.[6] Figure 2 shows the relationship between the sets of nations, subsidiaries, plants and processes. A nation may have multiple subsidiaries, and a subsidiary may have multiple plants. A plant includes its own production processes and material storage units; if it is a distribution center, it includes only the latter. Currency storage units and transportation processes are owned not by the plant but by the subsidiary. The definition of the currency set R is very important, and currencies in different subsidiaries are considered to be distinct members of the set, even if they are the same currency. For example, US dollar units in the USA and in Korea are distinct currency set members; that is, the exact meaning of an element of the set R is the "currency used in the subsidiary." Because multiple currencies can be used within a given subsidiary, the maximum number of members in a set R is equal to the product of the number of currencies and the number of subsidiaries. Of course, many nations use only one currency, so a superscript $r \in R$ is used to denote both a nation and a currency without loss of generality. Each production process requires multiple feedstock materials of fixed composition ($f_i^{j[\tau]}$), and each production process produces multiple products with a fixed product yield ($g_i^{j[\tau]}$), as shown in Figure 3a. If there are no material flows between a storage unit and a production process unit, the corresponding feedstock composition or product yield is zero. Each transportation process moves multiple parcels packing multiple materials of undetermined composition from storage units in a source plant to storage units in a destination plant, as shown in Figure 3b. The average material flow rate from storage $j \in J$ to storage $j' \in J$ via parcel $p \in P(n)$ in transportation process $n \in N$ at time period τ is denoted $D_{np}^{jj'[\tau]}$, where storage units j and j' store the same material. Note that transportation process n contains $P(n)$ parcels. Each storage unit contains one material, but the same material may be stored in multiple storage units located at different plants. As shown in Figure 4, each storage unit is associated with six types of material movement: purchasing from suppliers ($k \in K(j)$), shipping to consumers ($m \in M(j)$), feeding production processes, discharging from production processes, loading to transportation processes and unloading from transportation processes. The sets of suppliers $K(j)$ and consumers $M(j)$ are storage dependent. The material flow from a process to a storage unit (or from a storage unit to a process) is represented by the PSW model, as shown in Figure 2c of Ref. 10. In the PSW model, the material flow of a production process is represented in terms of four variables: batch size $B_i^{[\tau]}$, cycle time $\omega_i^{[\tau]}$, storage operation time fraction (SOTF) $\grave{x}_i^{[\tau]}$ (or $x_i'^{[\tau]}$), and startup time $\grave{t}_i^{[\tau]}$ (or $t_i'^{[\tau]}$). The SOTF $\grave{x}_i^{[\tau]}$ (or $x_i'^{[\tau]}$) is defined as the time required for material movement to (or from) the process, divided by the cycle time. The startup time $\grave{t}_i^{[\tau]}$ (or $\{t_i'^{[\tau]}$) is the first time at which the first batch is fed into (or discharged from) the process. It is assumed that the operations that feed feedstock into the process (or the operations that discharge product from the process) occur simultaneously, and that their SOTFs are the same. That is, the superscript j is not necessary to discriminate between the related storage units in $\grave{x}_i^{[\tau]}$ (or $x_i'^{[\tau]}$) and $\grave{t}_i^{[\tau]}$ (or $t_i'^{[\tau]}$). The above definitions apply to a parcel p in a transportation process n, and the corresponding notations are $B_{np}^{[\tau]}$, $\omega_n^{[\tau]}$, $\grave{x}_{np}^{[\tau]}$ (or $x_{np}'^{[\tau]}$), and $\grave{t}_{np}^{[\tau]}$ (or $t_{np}'^{[\tau]}$). The

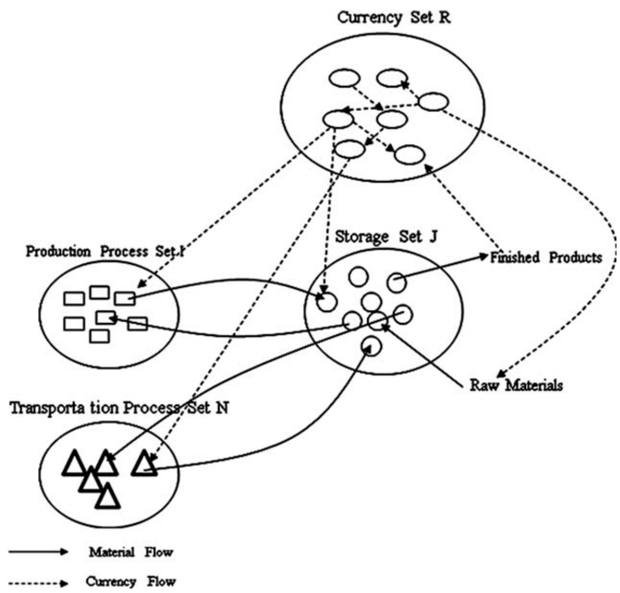

Figure 1. Structure of batch-storage network.

material flow of purchased raw material is represented by the order size $B_k^{j[\tau]}$, cycle time $\omega_k^{j[\tau]}$, SOTF $x_k^{j[\tau]}$, and startup time $t_k^{j[\tau]}$. All SOTFs are considered to be parameters, whereas the other variables are the design variables used in this study. The material flow of the finished product sales is represented by $B_m^{j[\tau]}, \omega_m^{j[\tau]}, x_m^{j[\tau]}$, and $t_m^{j[\tau]}$ in the same way. The arbitrarily periodic function of forecasted demand for the finished product is represented by a sum of PSW functions with known values of $B_m^{j[\tau]}, \omega_m^{j[\tau]}, x_m^{j[\tau]}$, and $t_m^{j[\tau]}$.[1] The general form of the PSW functions is defined as follows:

$$\mathrm{PSW}(t; D, \omega, t', x) = D\omega\left[\mathrm{int}\left[\frac{t-t'}{\omega}\right] + \min\left\{1, \frac{1}{x}\mathrm{res}\left[\frac{t-t'}{\omega}\right]\right\}\right] \tag{1}$$

or

$$\mathrm{PSW}'(t; B, \omega, t', x) = B\left[\mathrm{int}\left[\frac{t-t'}{\omega}\right] + \min\left\{1, \frac{1}{x}\mathrm{res}\left[\frac{t-t'}{\omega}\right]\right\}\right] \tag{2}$$

where D is the average flow rate, B is the batch size, ω is the cycle time, t' is the startup time, x is the SOTF, t is the current time, $\mathrm{int}[z]$ is the greatest integer less than or equal to z, and $\mathrm{res}[z] = z - \mathrm{int}[z]$. Note that $D = \frac{B}{\omega}$ and $\mathrm{PSW}'(t; g_i^{j[\tau]}B_i^{[\tau]}, \omega_i^{[\tau]}, t_i'^{[\tau]}), x_i'^{[\tau]})) = \int_0^t F_i^{[\tau]}(t)dt$, where $F_i^{[\tau]}(t)$ is given in Figure 2c of Ref. 10. Equation 1 is referred to the "first type" of PSW function, and Eq. 2 to the "second type" of PSW function. The average flow rate is used in the first type, whereas the batch size is used in the second type of PSW function. The two types of PSW function are associated with different upper and lower bounds and partial derivatives. Figure 3 of Ref. 10 shows the bounds on the second type of PSW function. Table 1 of Ref. 10 lists the expressions for the average and the upper and lower bounds of the first and second types of PSW functions,[10] where $\underline{\mathrm{PSW}} \le \mathrm{PSW} \le \overline{\mathrm{PSW}}$, $\underline{\mathrm{PSW}'} \le \mathrm{PSW}' \le \overline{\mathrm{PSW}'}$, $\overline{\mathrm{PSW}} = 0.5\left(\overline{\overline{\mathrm{PSW}}} + \underline{\underline{\mathrm{PSW}}}\right)$, and $\overline{\mathrm{PSW}'} = 0.5\left(\overline{\overline{\mathrm{PSW}'}} + \underline{\underline{\mathrm{PSW}'}}\right)$.

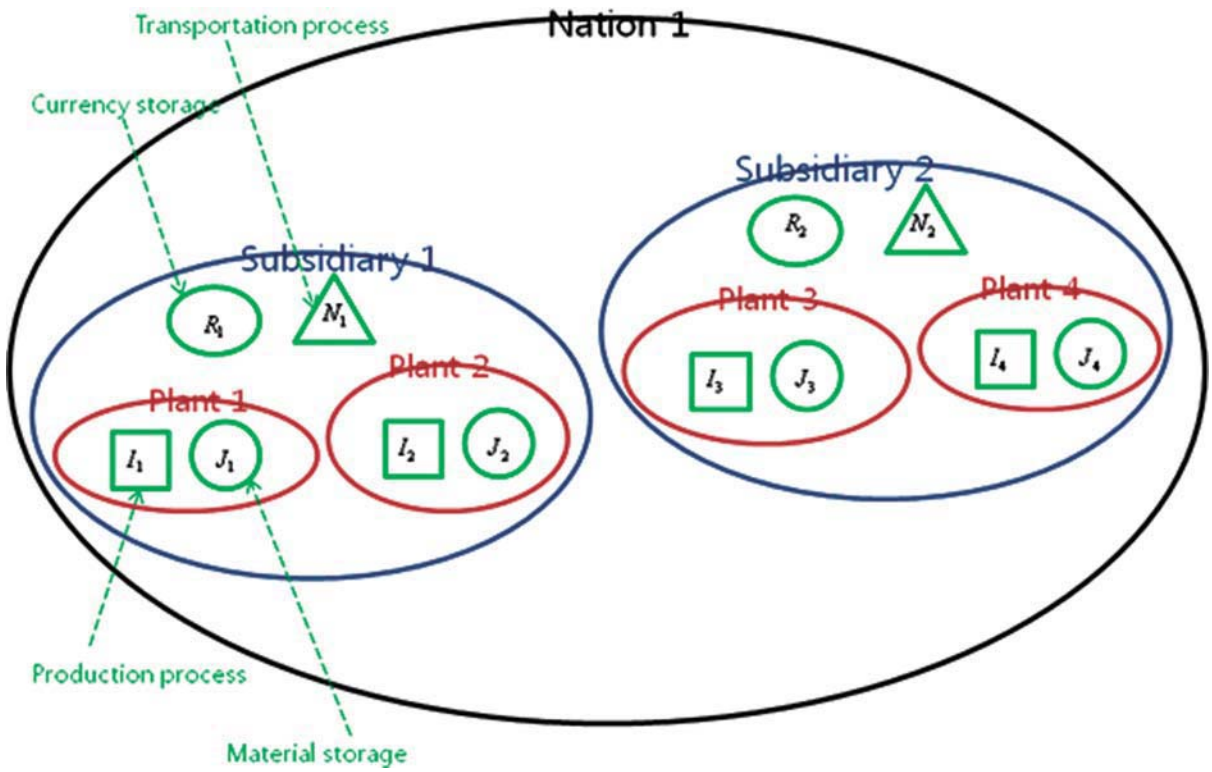

Figure 2. Relationship among sets.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

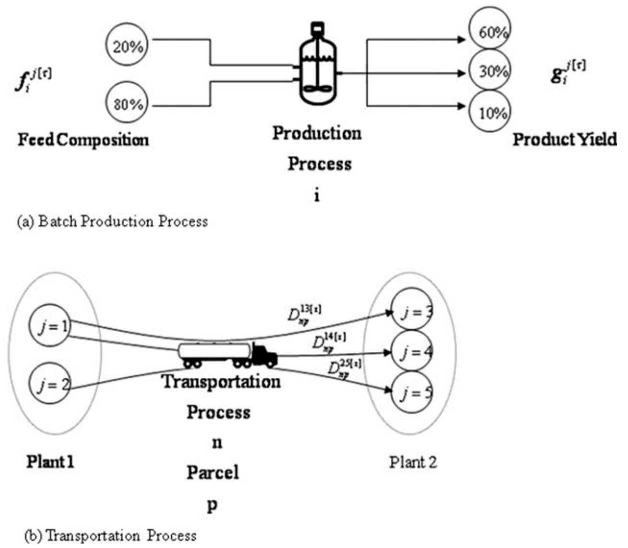

Figure 3. Production and transportation processes.

(a) Batch production process and (b) transportation process.

Let $D_i^{[\tau]}$ be the average material flow rate through the production process i. $D_i^{[\tau]}$ can be expressed as the batch size $B_i^{[\tau]}$ divided by the cycle time $\omega_i^{[\tau]}$. The average material flow rates through the transportation process n, raw material purchase from suppliers, and the shipping of the finished product to the consumers are described by $D_{np}^{[\tau]} (\equiv \sum_{j=1}^{|J|} \sum_{j'=1}^{|J|} D_{np}^{jj'[\tau]}), D_k^{j[\tau]}$, and $D_m^{j[\tau]}$, respectively, where $D_{np}^{[\tau]} = \frac{B_{np}^{[\tau]}}{\omega_n^{[\tau]}}$, $D_k^{j[\tau]} = \frac{B_k^{j[\tau]}}{\omega_k^{j[\tau]}}$, and $D_m^{j[\tau]} = \frac{B_m^{j[\tau]}}{\omega_m^{j[\tau]}}$.

The purchasing setup cost of the raw material j, paid in currency r, is denoted by $A_k^{jr[\tau]}$ (currency/order), the setup cost of the production process i, paid in currency r, is denoted by $A_i^{r[\tau]}$ (currency/batch), and the setup cost of the parcel p in transportation process n, paid in currency r, is denoted by $A_{np}^{r[\tau]}$ (currency/batch). The aggregated input data can be given to $\sum_{p=1}^{|P(n)|} A_{np}^{r[\tau]}$. The annual inventory holding cost for material storage j, paid in currency r, is denoted by $H^{jr[\tau]}$ (currency/L/year). Note that all material flows are measured volumetrically for convenience. The inventory holding cost is further segregated into the inventory operating cost ($h^{jr[\tau]}$) and the opportunity cost of inventory holding ($\gamma^{jr[\tau]}$); that is, $H^{jr[\tau]} = h^{jr[\tau]} + \gamma^{jr[\tau]}$. The operating cost

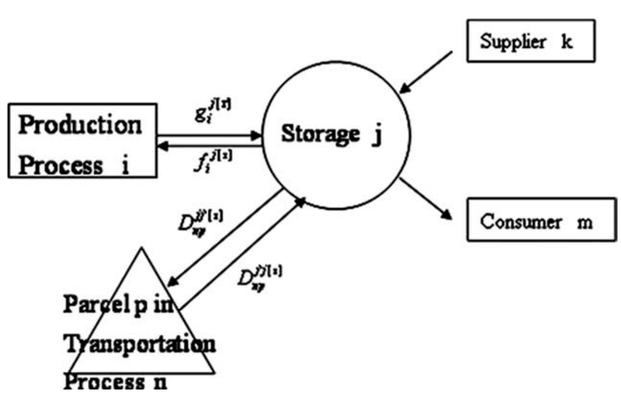

Figure 4. Input output streams of a storage unit.

Table 1. Mathematical Representation of Currency Flows

	PSW Flows
CF1	$\sum_{j=1}^{\|J\|} \sum_{m=1}^{\|M(j)\|} P_m^{jr[\tau]} \mathrm{PSW}(t; D_m^{j[\tau]}, \omega_m^{j[\tau]}, t_m^{j[\tau]} + \Delta t_m^{j[\tau]}, x_m^{j[\tau]})$
CF2	$\sum_{r' \neq r}^{\|R\|} \chi^{r'r[\tau]} \mathrm{PSW}(t; E^{r'r[\tau]}, \omega^{r'r[\tau]}, t^{r'r[\tau]}, 0)$
CF3	$-\sum_{j=1}^{\|J\|} \sum_{k=1}^{\|K(j)\|} P_k^{jr[\tau]} \mathrm{PSW}(t; D_k^{j[\tau]}, \omega_k^{j[\tau]}, t_k^{j[\tau]} + \Delta t_k^{j[\tau]}, x_k^{j[\tau]})$
CF4	$-\sum_{r' \neq r}^{\|R\|} \mathrm{PSW}(t; E^{rr'[\tau]}, \omega^{rr'[\tau]}, t^{rr'[\tau]}, 0)$
CF5	$-\sum_{j=1}^{\|J\|} \sum_{k \in \{D_k^{j[z]}\}^+}^{\|K(j)\|} \mathrm{PSW}'(t; A_k^{jr[\tau]}, \omega_k^{j[\tau]}, t_k^{j[\tau]}, x_k^{j[\tau]})$
CF6	$-\sum_{i \in \{D_i^{[z]}\}^+}^{\|I\|} \mathrm{PSW}'(t; A_i^{r[\tau]}, \omega_i^{[\tau]}, \grave{t}_i^{[z]}, \grave{x}_i^{[z]})$
CF7	$-\sum_{n \in \{D_{np}^{[\tau]}\}^+}^{\|N\|} \sum_{\mathrm{P} \in \{D_{np}^{[\tau]}\}^+}^{\|\mathrm{P}(n)\|} \mathrm{PSW}'(t; A_{np}^{r[\tau]}, \omega_n^{[\tau]}, t_{np}^{[\tau]}, \grave{x}_{np}^{[\tau]})$
CF8	$-\sum_{r' \in \{E^{rr'[\tau]}\}^+}^{\|R\|} \mathrm{PSW}'(t; A^{rr'[\tau]}, \omega^{rr'[\tau]}, t^{rr'[\tau]}, 0)$
CF9	$-\sum_{j=1}^{\|J\|} h^{jr[\tau]} \int_0^t V^{j[\tau]}(t) dt$
CF10	$-\sum_{o=1}^{\|O\|} E_o^{r[\tau]} t$
CF11	$-\sum_{j=1}^{\|J\|} \sum_{j' \neq j}^{\|J\|} \sum_{n=1}^{\|N\|} \sum_{p=1}^{\|\mathrm{P}(n)\|} \mathrm{PSW}(t; \pi_{np}^{j'jn[\tau]}, D_{np}^{j'j[\tau]}, \omega_n^{[\tau]}, t'^{[\tau]}_n, x'^{[\tau]}_n)$
CF12	$-\sum_{i=1}^{\|I\|} \mathrm{PSW}(t; \pi_i^{r[\tau]} D_i^{[\tau]}, \omega_i^{[\tau]}, \grave{t}_i^{[z]}, \grave{x}_i^{[z]})$
CF13	$-\sum_{j=1}^{\|J\|} \sum_{k=1}^{\|K(j)\|} a_k^{jr[\tau]} D_k^{j[\tau]} \omega_k^{j[\tau]} t$
CF14	$-\sum_{i=1}^{\|I\|} a_i^{r[\tau]} D_i^{[\tau]} \omega_i^{[\tau]} t$
CF15	$-\sum_{n=1}^{\|N\|} a_n^{r[\tau]} \sum_{j=1}^{\|J\|} \sum_{j' \neq j}^{\|J\|} D_n^{jj'[\tau]} \omega_n^{[\tau]} t$
CF16	$-\sum_{j=1}^{\|J\|} b^{jr[\tau]} \overline{\overline{V^{j[\tau]}}} t$

accompanies actual currency flow but the opportunity cost does not. The operating cost is involved in both constraint and objective function whereas the opportunity cost is involved in only objective function. To obtain an analytical solution to this optimization problem, the capital cost is assumed to be proportional to the processing capacity. Suppose that $a_k^{jr[\tau]}$ (currency/L/year) is the annual capital cost per unit capacity of the purchasing facility for the raw material j, paid in currency r, $a_i^{r[\tau]}$ (currency/L/year) is the annual capital cost per unit capacity of production process i, paid in currency r, $a_n^{r[\tau]}$ (currency/L/year) is the annual capital cost per unit capacity of transportation process n, paid in currency r, and $b^{jr[\tau]}$ (currency/L/year) is the annual capital cost per unit capacity of storage j, paid in currency r. In addition, assume that the raw material cost is proportional to the quantity and purchase price of raw material j from supplier k, paid in currency r is $P_k^{jr[\tau]}$ (currency/L). The sales price of the finished product j to consumer m, paid in currency r, is $P_m^{jr[\tau]}$ (currency/L).

Given that one production cycle in a production process is composed of the feedstock feeding time ($\grave{x}_i^{[\tau]} \omega_i^{[\tau]}$), processing time $((1 - \grave{x}_i^{[\tau]} - x_i'^{[\tau]})) \omega_i^{[\tau]})$, and product discharge time

$(x_i'^{[\tau]})\omega_i^{[\tau]}$), the timing relationship between the startup time of the feedstock streams and the startup time of the product streams is

$$t_i'^{[\tau]} = \grave{t}_i^{[\tau]} + \Delta t_i^{[\tau]}(.) \qquad \forall i, \tau \tag{3}$$

where $\Delta t_i^{[\tau]}(.)$ is an arbitrary function of the cycle time, batch size, and SOFT, for example, $\omega_i^{[\tau]}(1 - x_i'^{[\tau]})$, and is usually smaller than the cycle time. A similar equation holds for transportation processes.

$$\grave{t}_{np}^{[\tau]} = \grave{t}_n^{[\tau]} + \omega_n^{[\tau]}\grave{y}_{np}^{[\tau]},\; t'^{[\tau]}_{np} = \grave{t}_n^{[\tau]} + \omega_n^{[\tau]} y'^{[\tau]}_{np} + \Delta t_{np}^{[\tau]}(.) \qquad \forall n, p, \tau \tag{4}$$

where $\grave{y}_{np}^{[\tau]}$ and $y'^{[\tau]}_{np}$ are parameters to define the loading and unloading sequence in linear relationship with cycle time. $\Delta t_{np}^{[\tau]}(.)$ is usually constant and can be larger than the cycle time. The overall material balance associated with the material storage yields the following multiperiod material inventory replenishment relationship:

$$\begin{aligned}\dot{v}^{j[\tau]} = {} & \sum_{i=1}^{|I|} g_i^{j[\tau]} D_i^{[\tau]} + \sum_{k=1}^{|K(j)|} D_k^{j[\tau]} - \sum_{i=1}^{|I|} f_i^{j[\tau]} D_i^{[\tau]} - \sum_{m=1}^{|M(j)|} D_m^{j[\tau]} \\ & + \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} D_{np}^{j'j[\tau]} - \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} D_{np}^{jj'[\tau]} \\ & v^{j[\tau+1]} = v^{j[\tau]} + \dot{v}^{j[\tau]}\nabla t^{[\tau]},\; v^{j[0]} = v_0^j \qquad \forall j, \tau\end{aligned} \tag{5}$$

where $\dot{v}^{j[\tau]}$ is the material imbalance rate at time period τ, and $v^{j[\tau]} \geq 0$ is the multiperiod material inventory level at time period τ. The multiperiod inventory level is computed at discrete time points and does not reflect the actual operation of the various processes. The initial inventory in material storage j at time period τ is denoted by $V^{j[\tau]}(0)$, and the inventory held in storage j in the time period τ at time t is denoted by $V^{j[\tau]}(t)$ where $0 \leq t \leq \nabla t^{[\tau]}$. A material storage unit is connected to the incoming flows from the suppliers and production/transportation processes, and the outgoing flows to the consumers and production/transportation processes. The resulting inventory holding function for a material storage unit is

$$\begin{aligned}V^{j[\tau]}(t) = {} & V^{j[\tau]}(0) + \sum_{k=1}^{|K(j)|} \mathrm{PSW}(t; D_k^{j[\tau]}, \omega_k^{j[\tau]}, t_k^{j[\tau]}, x_k^{j[\tau]}) \\ & + \sum_{i=1}^{|I|} \mathrm{PSW}(t; g_i^{j[\tau]} D_i^{[\tau]}, \omega_i^{[\tau]}, t_i'^{[\tau]}, x_i'^{[\tau]}) \\ & - \sum_{i=1}^{|I|} \mathrm{PSW}(t; f_i^{j[\tau]} D_i^{[\tau]}, \omega_i^{[\tau]}, \grave{t}_i^{[\tau]}, \grave{x}_i^{[\tau]}) \\ & - \sum_{m=1}^{|M(j)|} \mathrm{PSW}(t; D_m^{j[\tau]}, \omega_m^{j[\tau]}, t_m^{j[\tau]}, x_m^{j[\tau]}) \\ & - \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} \mathrm{PSW}(t; D_{np}^{jj'[\tau]}, \omega_n^{[\tau]}, \grave{t}_{np}^{[\tau]}, \grave{x}_{np}^{[\tau]}) \\ & + \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} \mathrm{PSW}(t; D_{np}^{j'j[\tau]}, \omega_n^{[\tau]}, t'^{[\tau]}_{np}, x'^{[\tau]}_{np}) \quad \forall j, \tau\end{aligned} \tag{6}$$

To calculate the upper bound of the inventory holding, the positive PSW(.) terms in Eq. 6 are replaced with $\overline{\overline{\mathrm{PSW}}}(.)$ term and the negative terms in Eq. 6 are replaced with $\underline{\underline{\mathrm{PSW}}}(.)$ term.[10] The lower bound of the inventory holding is calculated in vice versa. The functional shape of $\overline{\overline{\mathrm{PSW}}}(.)$ and $\underline{\underline{\mathrm{PSW}}}(.)$ is given in Table 1 of Ref. 10. Equation 5 eliminates many terms and the resulting equations are

$$\begin{aligned}\overline{\overline{V^{j[\tau]}}} = {} & v^{j[\tau]} + \max\left\{0, \dot{v}^{j[\tau]}\nabla t^{[\tau]}\right\} + \nabla v^{j[\tau]} + \sum_{k=1}^{|K(j)|}(1 - x_k^{j[\tau]}) D_k^{j[\tau]} \omega_k^{j[\tau]} \\ & - \sum_{k=1}^{|K(j)|} D_k^{j[\tau]} t_k^{j[\tau]} + \sum_{i=1}^{|I|}(1 - x_i'^{[\tau]}) g_i^{j[\tau]} D_i^{[\tau]} \omega_i^{[\tau]} - \sum_{i=1}^{|I|} g_i^{j[\tau]} D_i t_i'^{[\tau]} \\ & + \sum_{i=1}^{|I|} f_i^{j[\tau]} D_i^{[\tau]} \grave{t}_i^{[\tau]} + \sum_{m=1}^{|M(j)|} D_m^{j[\tau]} t_m^{j[\tau]} + \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|}(1 - x'^{[\tau]}_{np}) D_{np}^{j'j[\tau]} \omega_n^{[\tau]} \\ & - \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} D_{np}^{j'j[\tau]} t'^{[\tau]}_{np} + \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} D_{np}^{jj'[\tau]} \grave{t}_{np}^{[\tau]} \quad \forall j, \tau\end{aligned} \tag{7}$$

$$\begin{aligned}\underline{\underline{V^{j[\tau]}}} = {} & v^{j[\tau]} + \min\left\{0, \dot{v}^{j[\tau]}\nabla t^{[\tau]}\right\} + \nabla v^{j[\tau]} - \sum_{k=1}^{|K(j)|} D_k^{j[\tau]} t_k^{j[\tau]} \\ & - \sum_{i=1}^{|I|} g_i^{j[\tau]} D_i^{[\tau]} t_i'^{[\tau]} - \sum_{i=1}^{|I|}(1 - \grave{x}_i^{[\tau]}) f_i^{j[\tau]} D_i^{[\tau]} \omega_i^{[\tau]} + \sum_{i=1}^{|I|} f_i^{j[\tau]} D_i^{[\tau]} \grave{t}_i^{[\tau]} \\ & - \sum_{m=1}^{|M(j)|}(1 - x_m^{j[\tau]}) D_m^{j[\tau]} \omega_m^{j[\tau]} + \sum_{m=1}^{|M(j)|} D_m^{j[\tau]} t_m^{j[\tau]} - \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} D_{np}^{j'j[\tau]} t'^{[\tau]}_{np} \\ & - \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|}(1 - \grave{x}_{np}^{[\tau]}) D_{np}^{jj'[\tau]} \omega_n^{[\tau]} + \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} D_{np}^{jj'[\tau]} \grave{t}_{np}^{[\tau]} \quad \forall j, \tau\end{aligned} \tag{8}$$

The average value of the inventory holding is calculated by replacing the PSW terms in Eq. 6 with $\overline{\mathrm{PSW}}$ term in Table 1 of Ref. 10. Equation 5 eliminates many terms and the resulting equations are

$$\begin{aligned}\overline{V^{j[\tau]}} = {} & v^{j[\tau]} + 0.5\dot{v}^{j[\tau]}\nabla t^{[\tau]} + \nabla v^{j[\tau]} + \sum_{k=1}^{|K(j)|}\frac{(1 - x_k^{j[\tau]})}{2} D_k^{j[\tau]} \omega_k^{j[\tau]} \\ & - \sum_{k=1}^{|K(j)|} D_k^{j[\tau]} t_k^{j[\tau]} + \sum_{i=1}^{|I|}\frac{(1 - x_i'^{[\tau]})}{2} g_i^{j[\tau]} D_i^{[\tau]} \omega_i^{[\tau]} - \sum_{i=1}^{|I|} g_i^{j[\tau]} D_i^{[\tau]} t_i'^{[\tau]} \\ & - \sum_{i=1}^{|I|}\frac{(1 - \grave{x}_i^{[\tau]})}{2} f_i^{j[\tau]} D_i^{[\tau]} \omega_i^{[\tau]} + \sum_{i=1}^{|I|} f_i^{j[\tau]} D_i^{[\tau]} \grave{t}_i^{[\tau]} - \sum_{m=1}^{|M(j)|}\frac{(1 - x_m^{j[\tau]})}{2} D_m^{j[\tau]} \omega_m^{j[\tau]} \\ & + \sum_{m=1}^{|M(j)|} D_m^{j[\tau]} t_m^{j[\tau]} + \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|}\frac{(1 - x'^{[\tau]}_{np})}{2} D_{np}^{j'j[\tau]} \omega_n^{[\tau]} \\ & - \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} D_{np}^{j'j[\tau]} t'^{[\tau]}_{np} - \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|}\frac{(1 - \grave{x}_{np}^{[\tau]})}{2} D_{np}^{jj'[\tau]} \omega_n^{[\tau]} \\ & + \sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\sum_{j'\neq j}^{|J|} D_{np}^{jj'[\tau]} \grave{t}_{np}^{[\tau]} \qquad \forall j, \tau\end{aligned} \tag{9}$$

where $\nabla v^{j[\tau]}$, the difference between the initial material inventory and the multiperiod material inventory, is computed by $\nabla v^{r[\tau+1]} + v^{j[\tau+1]} = V^{r[\tau]}(\nabla t^{[\tau]}) = V^{r[\tau+1]}(0)$. Equation 7 is used to predict the storage size, Eq. 8 is used to implement the no-depletion constraint, and Eq. 9 is used to calculate the inventory holding cost.

Suppose that there exist currency storage units of the type shown in Figure 4 of Ref. 10 that, through financial transactions, operate on a supply chain consisting of the batch process set I, transportation process set N, and material storage unit set J, as depicted in Figure 1. Let O, with subscript o, represent the set of stockholders. Corporate income tax is usually proportional to the gross profit and, thus, without loss of generality, is described as a payment to a fictitious stockholder $\bar{\boldsymbol{o}} \in \boldsymbol{O}$. We ignore the effects of temporary financial investments, bank loans, sales tax, and labor costs, which were treated in the previous study,[4,5,10] for simplicity. The currency flows (CF) entering the currency storage r are as follows:

1. CF1: Collection of accounts receivable after a collection drifting time $\Delta t_m^{j[\tau]}$ (year) from the shipping of the finished product to consumer m.
2. CF2: Currency transfer from currency storage r' to currency storage r with exchange rate $\chi^{r'r[\tau]}$ (currency/currency).

The currency flows leaving the currency storage r are as follows:

3. CF3: Disbursement of accounts payable after the disbursement drifting time $\Delta t_k^{j[\tau]}$ (year) for raw materials purchase from supplier k.
4. CF4: Currency transfer from currency storage r to currency storage r' with the exchange rate $\chi^{rr'[\tau]}$.
5. CF5: Purchase setup costs $A_k^{jr[\tau]}$ (currency/transaction).
6. CF6: Production process setup costs $A_i^{r[\tau]}$ (currency/transaction).
7. CF7: Transportation process setup costs $A_{np}^{r[\tau]}$ (currency/transaction).
8. CF8: Outgoing currency transfer setup costs $A^{rr'[\tau]}$ (currency/transaction).
9. CF9: Inventory operating costs $h^{jr[\tau]}$ (currency/L/year).
10. CF10: Dividends to stockholders, which include the corporate income tax, paid in currency r to the internal revenue service (IRS) at a rate $\xi^{r[\tau]}$ (currency/currency).
11. CF11: Customs duty payment proportional to the material flow rate from the material storage j' to the material storage j, at a rate $\pi_{np}^{j'jr[\tau]}$ (currency/L).
12. CF12: Variable operating costs for production processes, proportional to the material flow rate $\pi_i^{r[\tau]}$ (currency/L)
13. CF13: Annualized capital investment costs of purchasing equipment proportional to equipment capacity $a_k^{jr[\tau]}$ (currency/L/year)
14. CF14: Annualized capital investment costs of production equipment proportional to equipment capacity $a_i^{r[\tau]}$ (currency/L/year)
15. CF15: Annualized capital investment costs of transportation equipment proportional to equipment capacity $a_n^{r[\tau]}$ (currency/L/year)
16. CF16: Annualized capital investment costs of material storage proportional to equipment capacity $b^{jr[\tau]}$ (currency/L/year)

The currency is transferred between currency storage units, (CF2, CF4, and CF8). We denote the average currency flow rate $E^{r'r[\tau]}$ (currency/year), currency transfer cycle time $\omega^{r'r[\tau]}$, currency transfer startup time $t^{r'r[\tau]}$, from currency storage r' to currency storage r, with the exchange rate $\chi^{r'r[\tau]}$. The SOTF for currency flow is set to zero without loss of generality. The corresponding transfer setup costs $A^{r'r[\tau]}$ (currency/transaction) are paid from currency storage r'. We consider only the currency transfer between subsidiaries that results from material flows. For materials transported from storage $j' \in J(r')$ to storage $j \in J(r)$, with an average flow rate of $\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|} D_{np}^{j'j[\tau]}$, the purchasing costs will be transferred from the currency storage $r \in R(j)$ to the currency storage $r' \in R(j')$ based on the transfer price $P_{j'j}^{rr'[\tau]}$, where $J(r)$ is the subset of material storage associated with currency storage r, and $R(j)$ is the subset of currency storage associated with material storage j. The maximum currency transfer rate between subsidiaries is

$$\overline{\overline{E^{rr'[\tau]}}} = \sum_{j=1}^{|J|}\sum_{j' \neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|} P_{j'j}^{rr'[\tau]} D_{np}^{j'j[\tau]} \tag{10}$$

The actual currency transfer rate will be less than the estimate described by Eq. 10 due to reversal of material flows between subsidiaries. The equal quantities of currency that flow between subsidiaries in the opposite direction mutually counterbalance within a fixed time interval, for example, a period $\nabla t^{[\tau]}$. The actual currency transfer rate is

$$E^{rr'[\tau]} = \max\left\{0, \chi^{r1[\tau]}\overline{\overline{E^{rr'[\tau]}}} - \chi^{r'1[\tau]}\overline{\overline{E^{r'r[\tau]}}}\right\}\chi^{1r[\tau]} \tag{11}$$

Note that the transfer price $P_{j'j}^{rr'[\tau]} = 0$, if $r = r'$, $j = j'$, $j \notin J(r)$, $j' \notin J(r')$, $r \notin R(j)$ or $r' \notin R(j')$. Determination of the transfer price is very important and nontrivial work for MNCs. Transfer prices are related to the distribution of total benefits between the subsidiaries[20] and the minimization of total taxes.[21] In this study, we assume that the transfer price is given.

In addition, we assume that the setup cost transactions of CF5–CF8 and the inventory operating costs of CF9 are paid on a pro rata basis as a function of the extent of material processing or the volume of the financial transactions. In other words, the currency flows associated with the setup cost transactions and their material or currency flows have the same cycle time, startup time, and SOTF, but different batch size. The currency flows associated with the inventory operating costs are proportional to the inventory level.

The average currency flow rate for customs duties on the material moved from material storage j' to material storage j, transported by parcel p in transportation process n and paid to the nation that uses currency r, is $\pi_{np}^{j'jr[\tau]} D_{np}^{j'j[\tau]}$, where $\pi_{np}^{j'jr[\tau]}$ (currency/L) is the customs duty rate. The variable operating cost, which is the production process operating cost proportional to the average material flow rate through the process, can be treated in the same manner as the customs duty. Annualized capital investment costs CF13–CF16 were not treated as actual financial flows in previous studies,[4,5,10] but they are considered to be actual financial flows in the present study (the formulation results are the same.). Each currency flow in the PSW model is represented by functions of the batch size (or average flow rate), cycle time, startup time, and SOTF in the same way as the material flows are represented (with appropriate super- or sub-scripts).

The most important assumption underlying the multiperiod formulation is that $\bigtriangledown t^{[\tau]}$ should be sufficiently greater than all cycle times, and startup times, but it should take on the lowest values satisfying $\nabla \boldsymbol{t}^{[\tau]} >> \omega_{\boldsymbol{k}}^{\boldsymbol{j}[\tau]}, \omega_{\boldsymbol{i}}^{[\tau]}, \omega_{\boldsymbol{n}}^{[\tau]}, \omega_{\boldsymbol{m}}^{\boldsymbol{j}[\tau]}, \omega^{\boldsymbol{r}\boldsymbol{r}'[\tau]}$, $\boldsymbol{t}_{\boldsymbol{k}}^{\boldsymbol{j}[\tau]} + \Delta\boldsymbol{t}_{\boldsymbol{k}}^{\boldsymbol{j}[\tau]}, \boldsymbol{t}_{\boldsymbol{i}}^{[\tau]} + \Delta\boldsymbol{t}_{\boldsymbol{i}}^{[\tau]}$, and $\grave{\boldsymbol{t}}_{\boldsymbol{n}}^{[\tau]} + \omega_{\boldsymbol{n}}^{[\tau]}\boldsymbol{y}_{\boldsymbol{np}}'^{[\tau]} + \Delta\boldsymbol{t}_{\boldsymbol{np}}^{[\tau]}, \boldsymbol{t}^{\boldsymbol{r}\boldsymbol{r}'[\tau]}, \boldsymbol{t}_{\boldsymbol{m}}^{\boldsymbol{j}[\tau]} + \Delta\boldsymbol{t}_{\boldsymbol{m}}^{\boldsymbol{j}[\tau]}$ so that the expressions for the upper and lower bounds in Table 1 or Figure 3 of Ref. 10 are satisfactory. Note that if $\Delta\boldsymbol{t}_{\boldsymbol{k}}^{\boldsymbol{j}[\tau]}$, $\Delta\boldsymbol{t}_{\boldsymbol{n}}^{[\tau]}$, and $\Delta\boldsymbol{t}_{\boldsymbol{m}}^{\boldsymbol{j}[\tau]}$ correspond to a month, then $\bigtriangledown t^{[\tau]}$ is on the order of a year. This restricts the usage of the multiperiod planning model in this study to strategic or tactical planning in a conservative sense.

Nonlinear Optimization Model

To obtain an analytical solution, it is necessary to assume that stockholder dividends and corporate income taxes begin to be paid at $t_o^{r[\tau]} = 0$.[10] The startup times of currency flows of annualized capital investments are assumed to be zero, too. The currency flows of stockholder dividends and annualized capital investments are assumed to be continuous, and the SOTFs for these currency flows are set to one. Invoking these assumptions simplifies the currency flows of stockholder dividends and annualized capital investments given in the following equation. Table 1 lists the functional forms of CF1–CF16 obtained using Eqs. 1 and 2 with the defined variables and parameters. Define $C^{r[\tau]}(0)$ as the initial currency inventory and $C^{r[\tau]}(t)$ as the currency inventory at time t, where $0 \le t \le \nabla t^{[\tau]}$. Then, the currency inventory at time t is calculated by adding the incoming flows (CF1–CF2) to the initial currency inventory and subtracting the outgoing flows (CF3–CF16):

$$
\begin{aligned}
C^{r[\tau]}(t) = {} & C^{r[\tau]}(0) + \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} P_m^{jr[\tau]}\mathrm{PSW}(t; D_m^{j[\tau]}, \omega_m^{j[\tau]}, t_m^{j[\tau]} + \Delta t_m^{j[\tau]}, x_m^{j[\tau]}) - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} P_k^{jr[\tau]}\mathrm{PSW}(t; D_k^{j[\tau]}, \omega_k^{j[\tau]}, t_k^{j[\tau]} + \Delta t_k^{j[\tau]}, x_k^{j[\tau]}) \\
& - \sum_{j=1}^{|J|}\sum_{k\in\{D_k^{j[\tau]}\}^+}^{|K(j)|} \mathrm{PSW}'(t; A_k^{jr[\tau]}, \omega_k^{j[\tau]}, t_k^{j[\tau]}, x_k^{j[\tau]}) - \sum_{i\in\{D_i^{[\tau]}\}^+}^{|I|} \mathrm{PSW}'(t; A_i^{r[\tau]}, \omega_i^{[\tau]}, \grave{t}_i^{[\tau]}, \grave{x}_i^{[\tau]}) - \sum_{j=1}^{|J|} h^{jr[\tau]}\int_0^t V^{j[\tau]}(t)dt - \sum_{o=1}^{|O|} E_o^{r[\tau]} t \\
& + \sum_{r'\neq r}^{|R|} \chi^{r'r[\tau]}\mathrm{PSW}(t; E^{r'r[\tau]}, \omega^{r'r[\tau]}, t^{r'r[\tau]}, 0) - \sum_{i=1}^{|I|}\mathrm{PSW}(t; \pi_i^{r[\tau]}D_i^{[\tau]}, \omega_i^{[\tau]}, \grave{t}_i^{[\tau]}, \grave{x}_i^{[\tau]}) - \sum_{r'\neq r}^{|R|}\mathrm{PSW}(t; E^{rr'[\tau]}, \omega^{rr'[\tau]}, t^{rr'[\tau]}, 0) \\
& - \sum_{r'\in\{E^{rr'[\tau]}\}^+}^{|R|} \mathrm{PSW}'(t; A^{rr'[\tau]}, \omega^{rr'[\tau]}, t^{rr'[\tau]}, 0) - \sum_{n\in\{D_{np}^{[\tau]}\}^+}^{|N|}\sum_{p\in\{D_{np}^{[\tau]}\}^+}^{|P(n)|} \mathrm{PSW}'(t; A_{np}^{r[\tau]}, \omega_n^{[\tau]}, \grave{t}_{np}^{[\tau]}, \grave{x}_{np}^{[\tau]}) \\
& - \sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|} \mathrm{PSW}(t; \pi_{np}^{j'jr[\tau]}D_{np}^{j'j[\tau]}, \omega_n^{[\tau]}, t'^{[\tau]}_{np}, x'^{[\tau]}_{np}) - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} a_k^{jr[\tau]}D_k^{j[\tau]}\omega_k^{j[\tau]} t \\
& - \sum_{i=1}^{|I|} a_i^{r[\tau]}D_i^{[\tau]}\omega_i^{[\tau]} t - \sum_{n=1}^{|N|} a_n^{r[\tau]}D_n^{[\tau]}\omega_n^{[\tau]} t - \sum_{j=1}^{|J|} b^{jr[\tau]}\overline{\overline{V^{j[\tau]}}} t \quad \forall r, \tau
\end{aligned} \tag{12}
$$

where $\left\{D_k^{j[\tau]}\right\}^+ \equiv \left\{k|D_k^{j\tau]}>0\right\}$, $\left\{D_i^{[\tau]}\right\}^+ \equiv \left\{i|D_i^{[\tau]}>0\right\}$, $\left\{D_{np}^{[\tau]}\right\}^+ \equiv \left\{(n,p)|D_{np}^{[\tau]}>0\right\}$, and $\left\{E^{r'r[\tau]}\right\}^+ \equiv \left\{r\neq r'|E^{r'r[\tau]}>0\right\}$ are the index sets with positive average flow rates. The average flow rates of currency into and out of a currency storage unit satisfy the following multiperiod currency inventory replenishment equations:

$$
\begin{aligned}
\dot{c}^{r[\tau]} + \sum_{o=1}^{|O|} E_o^{r[\tau]} = {} & \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} P_m^{jr[\tau]}D_m^{j[\tau]} - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} P_k^{jr[\tau]}D_k^{j[\tau]} - \sum_{j=1}^{|J|}\sum_{k\in\{D_k^{j[\tau]}\}^+}^{|K(j)|} \frac{A_k^{jr[\tau]}}{\omega_k^{j[\tau]}} - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} a_k^{jr[\tau]}D_k^{j[\tau]}\omega_k^{j[\tau]} \\
& - \sum_{i\in\{D_i^{[\tau]}\}^+}^{|I|} \frac{A_i^{r[\tau]}}{\omega_i^{[\tau]}} - \sum_{i=1}^{|I|} a_i^{r[\tau]}D_i^{[\tau]}\omega_i^{[\tau]} - \sum_{i=1}^{|I|}\pi_i^{r[\tau]}D_i^{[\tau]} - \sum_{r'\in\{E^{rr'[\tau]}\}^+}^{|R|} \frac{A^{rr'[\tau]}}{\omega^{rr'[\tau]}} + \sum_{r'\neq r}^{|R|}\chi^{r'r[\tau]}E^{r'r[\tau]} - \sum_{r'\neq r}^{|R|} E^{rr'[\tau]} - \sum_{j=1}^{|J|} h^{jr[\tau]}\overline{V^{j[\tau]}} - \sum_{j=1}^{|J|} b^{jr[\tau]}\overline{\overline{V^{j[\tau]}}} \\
& - \sum_{n\in\{D_{np}^{[\tau]}\}^+}^{|N|} \frac{\sum_{p=1}^{|P(n)|} A_{np}^{r[\tau]}}{\omega_n^{[\tau]}} - \sum_{n=1}^{|N|} a_n^{r[\tau]}D_n^{[\tau]}\omega_n^{[\tau]} - \sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|} \pi_{np}^{j'jr[\tau]}D_{np}^{j'j[\tau]} \quad \forall r, \tau
\end{aligned} \tag{13}
$$

$$c^{r[\tau+1]} = c^{r[\tau]} + \dot{c}^{r[\tau]}\nabla t^{[\tau]}, c^{r[0]} = c_0^r \qquad \forall r, \tau \tag{14}$$

where $\dot{c}^{r[\tau]}$ is the currency imbalance rate over a time period τ, $c^{r[\tau]} \geq 0$ is the multiperiod currency inventory level during the time period τ, and $\nabla c^{r[\tau]}$, the difference between initial currency inventory and the multiperiod currency inventory, is computed by $c^{r[\tau+1]} + \nabla c^{r[\tau+1]} = C^{r[\tau]}(\nabla t^{[\tau]}) = C^{r[\tau+1]}(0)$.

Corporate income tax is computed based on the gross profit after capital investment cost (or depreciation) charges, $(E_{\bar{o}}^{r[\tau]}) = \xi^{r[\tau]}(\dot{c}^{r[\tau]} + \sum_{o=1}^{|O|} E_o^{r[\tau]})$, where $E_{\bar{o}}^{r[\tau]}$ is the average flow rate of the corporate income tax at a rate of $\xi^{r[\tau]}$ (currency/currency) paid in currency r. A nonprofitable subsidiary would not pay corporate income tax, and hence, $\xi^{r[\tau]} = 0$ for $r \in R(s)$ if $\sum_{r\in R(s)} E_{\bar{o}}^{r[\tau]} \leq 0$, where $R(s)$ is the subset of the currency storage owned by subsidiary s. A subsidiary of an MNC in a nation pays corporate income taxes according to that nation's tax regulations. Therefore, accounting must include the values of all materials moving into or out of the subsidiary. The right-hand side of Eq. 13 includes two currency transfer terms $\sum_{r'\neq r}^{|R|} \chi^{r'r[\tau]} E^{r'r[\tau]} - \sum_{r'\neq r}^{|R|} E^{rr'[\tau]}$. Note that $\sum_{r'\neq r}^{|R|} \chi^{r'r[\tau]} E^{r'r[\tau]} - \sum_{r'\neq r}^{|R|} E^{rr'[\tau]} = \sum_{r'\neq r}^{|R|} \chi^{r'r[\tau]} \overline{\overline{E^{r'r[\tau]}}} - \sum_{r'\neq r}^{|R|} \overline{\overline{E^{rr'[\tau]}}}$. The terms $\sum_{j=1}^{|J|} \sum_{k=1}^{|K(j)|} a_k^{jr[\tau]} D_k^{j[\tau]} \omega_k^{j[\tau]}$, $\sum_{i=1}^{|I|} a_i^{r[\tau]} D_i^{[\tau]} \omega_i^{[\tau]}$, $\sum_{n=1}^{|N|} a_n^{r[\tau]} D_n^{[\tau]} \omega_n^{[\tau]}$, and $\sum_{j=1}^{|J|} b^{jr[\tau]} \overline{\overline{Vj^{[\tau]}}}$ are the annualized capital investment costs or depreciations for the processes and storage units.

The average level of the currency inventory ($\overline{C^{r[\tau]}}$) is calculated by replacing the PSW(.) terms in Eq. 12 with $\overline{\text{PSW}}(.)$ term in Table 1 of Ref. 10. Equation 13 simplifies many terms and the resulting equation is

$$\begin{aligned}
\overline{C^{r[\tau]}} = {} & c^{r[\tau]} + 0.5\dot{c}^{r[\tau]}\nabla t^{[\tau]} + \nabla c^{r[\tau]} + \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} P_m^{jr[\tau]}\left[0.5(1-x_m^j)D_m^{j[\tau]}\omega_m^{j[\tau]} - D_m^{j[\tau]}\left(t_m^{j[\tau]} + \Delta t_m^{j[\tau]}\right)\right] \\
& - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} P_k^{jr[\tau]}\left[0.5(1-x_k^{j[\tau]})D_k^{j[\tau]}\omega_k^{j[\tau]} - D_k^{j[\tau]}\left(t_k^{j[\tau]} + \Delta t_k^{j[\tau]}\right)\right] - \sum_{i\in\left\{D_i^{[\tau]}\right\}^+}^{|I|} A_i^{r[\tau]}\left[0.5(1-\grave{x}_i^{[\tau]}) - \frac{\grave{t}_i^{[\tau]}}{\omega_i^{[\tau]}}\right] \\
& - \sum_{j=1}^{|J|}\sum_{k\in\left\{D_k^{j[\tau]}\right\}^+}^{|K(j)|} A_k^{jr[\tau]}\left[0.5(1-x_k^{j[\tau]}) - \frac{t_k^{j[\tau]}}{\omega_k^{j[\tau]}}\right] + \sum_{r'\neq r}^{|R|} \chi^{r'r[\tau]}\left[0.5E^{r'r[\tau]}\omega^{r'r[\tau]} - E^{r'r[\tau]}t^{r'r[\tau]}\right] - \sum_{r'\neq r}^{|R|}\left[0.5E^{rr'[\tau]}\omega^{rr'[\tau]} - E^{rr'[\tau]}t^{rr'[\tau]}\right] \\
& - \sum_{r'\in\left\{E^{rr'[\tau]}\right\}^+}^{|R|} A^{rr'[\tau]}\left[0.5 - \frac{t^{rr'[\tau]}}{\omega^{rr'[\tau]}}\right] - \sum_{i=1}^{|I|} \pi_i^{r[\tau]} D_i^{[\tau]}\left[0.5(1-\grave{x}_i^{[\tau]})\omega_i^{[\tau]} - \grave{t}_i^{[\tau]}\right] - \sum_{n\in\left\{D_{np}^{[\tau]}\right\}^+}^{|N|}\sum_{p\in\left\{D_{np}^{[\tau]}\right\}^+}^{|P(n)|} A_{np}^{r[\tau]}\left[0.5(1-\grave{x}_{np}^{[\tau]}) - \frac{\grave{t}_{np}^{[\tau]}}{\omega_n^{[\tau]}}\right] \\
& - \sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|} \pi_{np}^{j'jr[\tau]} D_{np}^{j'j[\tau]}\left[0.5(1-x'^{[\tau]}_{np})\omega_n^{[\tau]} - t'^{[\tau]}_{np}\right] \qquad \forall r, \tau
\end{aligned} \tag{15}$$

To calculate the lower bound of the currency inventory ($\underline{\underline{C^{r[\tau]}}}$), the negative PSW(.) terms in Eq. 12 are replaced with $\overline{\text{PSW}}(.)$ terms and the positive terms in Eq. 12 are replaced with $\underline{\text{PSW}}(.)$ terms.[10] The functional shape of $\overline{\text{PSW}}(.)$ and $\underline{\text{PSW}}(.)$ is given in Table 1 of Ref. 10. Equation 13 simplifies many terms and the resulting equation is

$$\begin{aligned}
\underline{\underline{C^{r[\tau]}}} = {} & c^{r[\tau]} + \min\left\{0, \dot{c}^{r[\tau]}\nabla t^{[\tau]}\right\} + \nabla c^{r[\tau]} - \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} P_m^{jr[\tau]}\left[D_m^{j[\tau]}\left(t_m^{j[\tau]} + \Delta t_m^{j[\tau]}\right)\right] - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} P_k^{jr[\tau]}\left[(1-x_k^{j[\tau]})D_k^{j[\tau]}\omega_k^{j[\tau]} - D_k^{j[\tau]}\left(t_k^{j[\tau]} + \Delta t_k^{j[\tau]}\right)\right] \\
& - \sum_{j=1}^{|J|} h^{jr[\tau]}\left(\overline{\overline{Vj^{[\tau]}}} - \overline{Vj^{[\tau]}}\right)\nabla t^{[\tau]} - \sum_{i\in\left\{D_i^{[\tau]}\right\}^+}^{|I|} A_i^{r[\tau]}\left[(1-\grave{x}_i^{[\tau]}) - \frac{\grave{t}_i^{[\tau]}}{\omega_i^{[\tau]}}\right] - \sum_{j=1}^{|J|}\sum_{k\in\left\{D_k^{j[\tau]}\right\}^+}^{|K(j)|} A_k^{jr[\tau]}\left[(1-x_k^{j[\tau]}) - \frac{t_k^{j[\tau]}}{\omega_k^{j[\tau]}}\right] - \sum_{r'\neq r}^{|R|} \chi^{r'r[\tau]} E^{r'r[\tau]} t^{r'r[\tau]} \\
& - \sum_{r'\neq r}^{|R|}\left[E^{rr'[\tau]}\omega^{rr'[\tau]} - E^{rr'[\tau]}t^{rr'[\tau]}\right] - \sum_{r'\in\left\{E^{rr'[\tau]}\right\}^+}^{|R|} A^{rr'[\tau]}\left(1 - \frac{t^{rr'[\tau]}}{\omega^{rr'[\tau]}}\right) - \sum_{i=1}^{|I|} \pi_i^{r[\tau]} D_i^{[\tau]}\left[(1-\grave{x}_i^{[\tau]})\omega_i^{[\tau]} - \grave{t}_i^{[\tau]}\right] \\
& - \sum_{n\in\left\{D_{np}^{[\tau]}\right\}^+}^{|N|}\sum_{p\in\left\{D_{np}^{[\tau]}\right\}^+}^{|P(n)|} A_{np}^{r[\tau]}\left[(1-\grave{x}_{np}^{[\tau]}) - \frac{\grave{t}_{np}^{[\tau]}}{\omega_n^{[\tau]}}\right] - \sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|} \pi_{np}^{j'jr[\tau]} D_{np}^{j'j[\tau]}\left[(1-x'^{[\tau]}_{np})\omega_n^{[\tau]} - t'^{[\tau]}_{np}\right] \qquad \forall r, \tau
\end{aligned} \tag{16}$$

Equation 15 is used to compute the opportunity cost of the currency inventory, and Eq. 16 should be nonnegative to ensure that the currency storage is not depleted, because a shortage of currency will incur severe additional costs or even bankruptcy. Therefore,

$$\underline{\underline{\underline{V^{j[\tau]}}}} \geq \min\left\{v^{j[\tau]}, v^{j[\tau+1]}\right\} \geq 0 \tag{17}$$

and

$$\underline{\underline{\underline{C^{r[\tau]}}}} \geq \min\left\{c^{r[\tau]}, c^{r[\tau+1]}\right\} \geq 0 \tag{18}$$

constitute constraints on the design optimization. Note that $\min\left\{v^{j[\tau]}, v^{j[\tau+1]}\right\} = v^{j[\tau]} + \min\left\{0, \dot{v}^{j[\tau]}\nabla t^{[\tau]}\right\}$ and $\min\left\{c^{r[\tau]}, c^{r[\tau+1]}\right\} = c^{r[\tau]} + \min\left\{0, \dot{c}^{r[\tau]}\nabla t^{[\tau]}\right\}$.

Suppose $\eta^{r[\tau]}$ (currency/currency/year) is the rate of the opportunity cost for the currency inventory (interest rate), $\rho^{r[\tau]}$ (currency/currency) is the discount rate for period τ used to compute the net present value, and $\gamma^{jr[T]}$ (currency/L/year) is the salvage value of the material j at the end of period T. The objective function of the optimization is to minimize the net present value of the opportunity costs of currency/material inventories minus the dividend to stockholders expressed in the numeraire currency ($r = 1$) over the entire finite time periods ($\tau = 0, 1, 2, ..., T$):

$$\text{Minimize } TC = \sum_{\tau=0}^{T-1} ATC^{[\tau]}\nabla t^{[\tau]} - \sum_{r=1}^{|R|}\sum_{j=1}^{|J|} \chi^{r1[T]}\gamma^{jr[T]}v^{j[T]} \tag{19}$$

where

$$ATC^{[\tau]} = \sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}\overline{C^{r[\tau]}} + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|} \tilde{\chi}^{r1[\tau]}\gamma^{jr[\tau]}\overline{V^{j[\tau]}} - \sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]}\left(\dot{c}^{r[\tau]} + \sum_{o\neq\bar{o}}^{|O|} E_o^{r[\tau]}\right) \quad \forall\tau \tag{20}$$

and $\tilde{\chi}^{r1[\tau]} \equiv \frac{\chi^{r1[\tau]}}{(1+\rho^{r[\tau]})^{[\tau]}}$. $\left(\dot{c}^{r[\tau]} + \sum_{o\neq\bar{o}}^{|O|} E_o^{r[\tau]}\right)$ can be developed further using Eq. 13 and $(E_{\bar{o}}^{r[\tau]}) = \xi^{r[\tau]}(\dot{c}^{r[\tau]} + \sum_{o=1}^{|O|} E_o^{r[\tau]})$.

$$\begin{aligned}
\dot{c}^{r[\tau]} + \sum_{o\neq\bar{o}}^{|O|} E_o^{r[\tau]} = {} & (1-\xi^{r[\tau]})\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} P_m^{jr[\tau]}D_m^{j[\tau]} - (1-\xi^{r[\tau]})\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} P_k^{jr[\tau]}D_k^{j[\tau]} - (1-\xi^{r[\tau]})\sum_{j=1}^{|J|}\sum_{k\in\left\{D_k^{j[z]}\right\}^+}^{|K(j)|} \frac{A_k^{jr[\tau]}}{\omega_k^{j[\tau]}} \\
& - (1-\xi^{r[\tau]})\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} a_k^{jr[\tau]}D_k^{j[\tau]}\omega_k^{j[\tau]} - (1-\xi^{r[\tau]})\sum_{i\in\left\{D_i^{[Z]}\right\}^+}^{|I|} \frac{A_i^{r[\tau]}}{\omega_i^{[\tau]}} - (1-\xi^{r[\tau]})\sum_{i=1}^{|I|} a_i^{r[\tau]}D_i^{[\tau]}\omega_i^{[\tau]} - (1-\xi^{r[\tau]})\sum_{i=1}^{|I|} \pi_i^{r[\tau]}D_i^{[\tau]} \\
& - (1-\xi^{r[\tau]})\sum_{j=1}^{|J|} h^{jr[\tau]}\overline{V^{j[\tau]}} - (1-\xi^{r[\tau]})\sum_{j=1}^{|J|} b^{jr[\tau]}\overline{\overline{V^{j[\tau]}}} - (1-\xi^{r[\tau]})\sum_{r'\in\left\{E^{rr'[\tau]}\right\}^+}^{|R|} \frac{A^{rr'[\tau]}}{\omega^{rr'[\tau]}} + (1-\xi^{r[\tau]})\sum_{r'\neq r}^{|R|} \chi^{r'r[\tau]}\overline{\overline{E^{r'r[\tau]}}} \\
& - (1-\xi^{r[\tau]})\sum_{r'\neq r}^{|R|} \overline{\overline{E^{rr'[\tau]}}} - (1-\xi^{r[\tau]})\sum_{n\in\left\{D_{np}^{[\tau]}\right\}^+}^{|N|} \frac{\sum_{p=1}^{|P(n)|} A_{np}^{r[\tau]}}{\omega_n^{[\tau]}} - (1-\xi^{r[\tau]})\sum_{n=1}^{|N|} a_n^{r[\tau]}D_n^{[\tau]}\omega_n^{[\tau]} - (1-\xi^{r[\tau]})\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|} \pi_{np}^{j'jr[\tau]}D_{np}^{j'j[\tau]} \quad \forall r,\tau
\end{aligned} \tag{21}$$

Equation 20 can be rewritten as

$$\begin{aligned}
ATC^{[\tau]} = {} & \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} \tilde{\chi}^{r1[\tau]}\left[(1-\xi^{r[\tau]})\frac{A_k^{jr[\tau]}}{\omega_k^{j[\tau]}} + (1-\xi^{r[\tau]})a_k^{jr[\tau]}D_k^{j[\tau]}\omega_k^{j[\tau]} + (1-\xi^{r[\tau]})P_k^{jr[\tau]}D_k^{j[\tau]}\right] \\
& + \sum_{r=1}^{|R|}\sum_{i=1}^{|I|} \tilde{\chi}^{r1[\tau]}\left[(1-\xi^{r[\tau]})\frac{A_i^{r[\tau]}}{\omega_i^{[\tau]}} + (1-\xi^{r[\tau]})a_i^{r[\tau]}D_i^{[\tau]}\omega_i^{[\tau]} + (1-\xi^{r[\tau]})\pi_i^{r[\tau]}D_i^{[\tau]}\right] \\
& + \sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|} \tilde{\chi}^{r1[\tau]}(1-\xi^{r[\tau]})\frac{A^{rr'[\tau]}}{\omega^{rr'[\tau]}} - \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} \tilde{\chi}^{r1[\tau]}(1-\xi^{r[\tau]})P_m^{jr[\tau]}D_m^{j[\tau]} \\
& + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|} \tilde{\chi}^{r1[\tau]}\left[\left((1-\xi^{r[\tau]})h^{jr[\tau]} + \gamma^{jr[\tau]}\right)\overline{V^{j[\tau]}} + (1-\xi^{r[\tau]})b^{jr[\tau]}\overline{\overline{V^{j[\tau]}}}\right] \\
& + \sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}\overline{C^{r[\tau]}} + \sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|} \tilde{\chi}^{r1[\tau]}(1-\xi^{r[\tau]})\overline{\overline{E^{rr'[\tau]}}} - \sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|} \tilde{\chi}^{r'1[\tau]}(1-\xi^{r'[\tau]})\chi^{rr'[\tau]}\overline{\overline{E^{rr'[\tau]}}} \\
& + \sum_{r=1}^{|R|}\sum_{n=1}^{|N|} \tilde{\chi}^{r1[\tau]}\left[(1-\xi^{r[\tau]})\frac{\sum_{p=1}^{|P(n)|} A_{np}^{r[\tau]}}{\omega_n^{[\tau]}} + (1-\xi^{r[\tau]})a_n^{r[\tau]}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{p=1}^{|P(n)|} D_{np}^{jj'[\tau]}\omega_n^{[\tau]} + (1-\xi^{r[\tau]})\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{p=1}^{|P(n)|} \pi_{np}^{j'jr[\tau]}D_{np}^{j'j[\tau]}\right] \quad \forall\tau
\end{aligned} \tag{22}$$

The independent variables are selected to be the cycle times ($\omega_k^{j[\tau]}$,$\omega_i^{[\tau]}$,$\omega_n^{[\tau]}$, and $\omega^{rr'[\tau]}$), start-up times ($t_k^{j[\tau]}$, $t_i^{[\tau]}$, $t_{np}^{[\tau]}$, and $t^{rr'[\tau]}$), and average material/currency flow rates ($D_k^{j[\tau]}$, $D_i^{[\tau]}$,$D_{np}^{jj'[\tau]}$, and $E_o^{r[\tau]}$). Note that the startup time $t_i'^{[\tau]}$), $\grave{\boldsymbol{t}}_{\boldsymbol{np}}$, and $t'^{[\tau]}_{np}$ are converted into $\grave{\boldsymbol{t}}_{\boldsymbol{i}}^{[\tau]}$ and $\grave{\boldsymbol{t}}_{\boldsymbol{n}}^{[\tau]}$ by Eqs. 3 and 4, respectively.

The objective function in Eq. 22 is convex, and the constraints Eqs. 17 and 18 are linear with respect to $\omega_k^{j[\tau]}$, $\omega_i^{[\tau]}$,$\omega_n^{[\tau]}$,$\omega^{rr'[\tau]}$, $t_k^{j[\tau]}$,$\grave{t}_i^{[\tau]}$,$\grave{t}_n^{[\tau]}$, and $t^{rr'[\tau]}$ if $D_k^{j[\tau]}$,$D_i^{[\tau]}$,$D_{np}^{jj'[\tau]}$, and $E_o^{r[\tau]}$ are considered to be parameters. However, the convexity with respect to $D_k^{j[\tau]}$,$D_i^{[\tau]}$,$D_{np}^{jj'[\tau]}$, and $E_o^{r[\tau]}$ is not clear. The solutions to the Kuhn-Tucker conditions, with respect to $\omega_k^{j[\tau]}$,$\omega_i^{[\tau]}$,$\omega_n^{[\tau]}$,$\omega^{rr'[\tau]}$, $t_k^{j[\tau]}$,$\grave{t}_i^{[\tau]}$,$\grave{t}_n^{[\tau]}$, and $t^{rr'[\tau]}$, are obtained when $D_k^{j[\tau]}$,$D_i^{[\tau]}$,$D_{np}^{jj'[\tau]}$, and $E_o^{r[\tau]}$ are considered to be parameters. The first-level problem entails

P1: Minimizing the objective function in Eqs. 19 and 22, under the constraints given in Eqs. 8, 9 and 15–18 with respect to the design variables $\omega_k^{j[\tau]}$,$\omega_i^{[\tau]}$,$\omega_n^{[\tau]}$,$\omega^{rr'[\tau]}$, $t_k^{j[\tau]}$, $\grave{t}_i^{[\tau]}$,$\grave{t}_n^{[\tau]}$, and $t^{rr'[\tau]}$.

The problem can then be solved with respect to $D_k^{j[\tau]}$,$D_i^{[\tau]}$,$D_{np}^{jj'[\tau]}$, and $E_o^{r[\tau]}$. Although the problem is thereby broken down into a two-level parametric optimization problem, the Kuhn-Tucker conditions for both the original and the two-level problem are the same if the constraints are reduced to equalities.[3] In other words, the Kuhn-Tucker conditions for the first-level problem produce an explicit analytical solution, and the original problem can be reduced to the second-level problem by eliminating the design variables in the first-level problem. The first-level problem within the two-level problem includes a convex objective function with linear inequality constraints, and the second-level problem includes a nonconvex objective function with nonlinear equality constraints. The two-level parametric approach yields a global optimum inasmuch as the second-level problem converges to its global optimum.

Solution to the Kuhn-Tucker Conditions

The solution to the Kuhn-Tucker conditions for the first-level optimization problem (P1) with fixed values of $D_k^{j[\tau]}$,$D_i^{[\tau]}$,$D_{np}^{jj'[\tau]}$, and $E_o^{r[\tau]}$, is obtained by means of the algebraic manipulations summarized in Supporting Information Appendix A posted in http://myweb.pknu.ac.kr/gbyi/. The optimal cycle times are

$$ {}^{*}\omega_k^{j[\tau]} = \sqrt{\frac{\left(\sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]} \mu^{r[\tau]} A_k^{jr[\tau]}\right)}{D_k^{j[\tau]}\left(\sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]} \Psi_k^{jr[\tau]}\right)}} \qquad \forall j,k,\tau \qquad (23) $$

$$ {}^{*}\omega_i^{[\tau]} = \sqrt{\frac{\left(\sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]} \mu^{r[\tau]} A_i^{r[\tau]}\right)}{D_i^{[\tau]}\left(\sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]} \Psi_i^{r[\tau]}\right)}} \qquad \forall i,\tau \qquad (24) $$

$$ {}^{*}\omega_n^{[\tau]} = \sqrt{\frac{\left(\sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]} \mu^{r[\tau]} \sum_{p=1}^{|P(n)|} A_{np}^{r[\tau]}\right)}{\left(\sum_{r=1}^{|R|} \tilde{\chi}^{r1[\tau]} \sum_{j=1}^{|J|} \sum_{j'\neq j}^{|J|} \sum_{p=1}^{|P(n)|} \Psi_{np}^{jj'r[\tau]} D_{np}^{jj'[\tau]}\right)}} \qquad \forall n,\tau \qquad (25) $$

$$ {}^{*}\omega^{rr'[\tau]} = \sqrt{\frac{\mu^{r[\tau]} A^{rr'[\tau]}}{E^{rr'[\tau]} \Psi^{rr'[\tau]}}} \qquad \forall r,r',\tau \qquad (26) $$

where

$$ \mu^{r[\tau]} \equiv \left(1-\xi^{r[\tau]}\right)\left(1-0.5\eta^{r[\tau]}\nabla t^{[\tau]}\right) \qquad \forall r,\tau \qquad (27) $$

$$ \theta^{jr[\tau]} \equiv \frac{\left(\mu^{r[\tau]}+\eta^{r[\tau]}\nabla t^{[\tau]}\right)h^{jr[\tau]}+\gamma^{jr[\tau]}}{2} + \mu^{r[\tau]} b^{jr[\tau]} \qquad \forall j,r,\tau \qquad (28) $$

$$ \Psi_k^{jr[\tau]} = \mu^{r[\tau]} a_k^{jr[\tau]} + \left[\frac{\eta^{r[\tau]} P_k^{jr[\tau]}}{2} + \theta^{jr[\tau]}\right](1-x_k^{j[\tau]}) \qquad \forall j,k,r,\tau \qquad (29) $$

$$ \Psi_i'^{r[\tau]} = \mu^{r[\tau]} a_i^{r[\tau]} + \left(1-\grave{x}_i^{[\tau]}\right)\sum_{j=1}^{|J|} \theta^{jr[\tau]} f_i^{j[\tau]} + \left(1-x_i'^{[\tau]}\right) \sum_{j=1}^{|J|} \theta^{jr[\tau]} g_i^{j[\tau]} + \left(1-\grave{x}_i^{[\tau]}\right)\sum_{j=1}^{|J|} \frac{\eta^{r[\tau]}\pi_i^{r[\tau]}}{2} \qquad \forall i,r,\tau \qquad (30) $$

$$ \Psi_{np}^{jj'r[\tau]} = \mu^{r[\tau]} a_n^{r[\tau]} + \left(1-\grave{x}_{np}^{[\tau]}\right)\theta^{jr[\tau]} + \left(1-x'^{[\tau]}_{np}\right)\theta^{j'r[\tau]} + \left(1-x'^{[\tau]}_{np}\right)\frac{\eta^{r[\tau]}\pi_{np}^{jj'r[\tau]}}{2} \qquad \forall n,r,j,j',\tau \qquad (31) $$

$$ \Psi^{rr'[\tau]} = 0.5\left(\left(\frac{\chi^{r'1[\tau]}}{\chi^{r1[\tau]}}\right)\eta^{r'[\tau]}\chi^{rr'[\tau]} + \eta^{r[\tau]}\right) \qquad \forall r,r',\tau \qquad (32) $$

Because the values of the multipliers are positive, Supporting Information Eq. A10 gives $\underline{\underline{V}}^{j[\tau]} - \left[v^{j[\tau]} + \min\left\{0, \dot{v}^{j[\tau]}\nabla t^{[\tau]}\right\}\right] = 0$ and $\underline{\underline{C}}^{r[\tau]} - \left[c^{r[\tau]} + \min\left\{0, \dot{c}^{r[\tau]}\nabla t^{[\tau]}\right\}\right] = 0$. Equations 8 and 16 yield the following expressions:

$$ \begin{aligned} &\sum_{k=1}^{|K(j)|} D_k^{j[\tau]} t_k^{j[\tau]} + \sum_{i=1}^{|I|}\left(g_i^{j[\tau]} - f_i^{j[\tau]}\right) D_i^{[\tau]} \grave{t}_i^{[\tau]} \\ &+ \sum_{n=1}^{|N|}\sum_{j'\neq j}^{|J|}\sum_{p=1}^{|P(n)|}\left(D_{np}^{j'j[\tau]} - D_{np}^{jj'[\tau]}\right)\grave{t}_n^{[\tau]} = \nabla v^{j[\tau]} \\ &- \sum_{m=1}^{|M(j)|}(1-x_m^{j[\tau]}) D_m^{j[\tau]} \omega_m^{j[\tau]} + \sum_{m=1}^{|M(j)|} D_m^{j[\tau]} t_m^{j[\tau]} - \sum_{i=1}^{|I|} g_i^{j[\tau]} D_i^{[\tau]} \Delta t_i^{[\tau]} \\ &- \sum_{i=1}^{|I|}(1-\grave{x}_i^{[\tau]}) f_i^{j[\tau]} D_i^{[\tau]}\left({}^{*}\omega_i^{[\tau]}\right) \\ &- \sum_{n=1}^{|N|}\sum_{j'\neq j}^{|J|}\sum_{p=1}^{|P(n)|} D_{np}^{j'j[\tau]}\left[\Delta t_{np}^{[\tau]} + \left({}^{*}\omega_n^{[\tau]}\right) y'^{[\tau]}_{np}\right] \\ &\qquad - \sum_{n=1}^{|N|}\sum_{j'\neq j}^{|J|}(1-\grave{x}_{np}^{[\tau]} - \grave{y}_{np}^{[\tau]}) D_{np}^{jj'[\tau]}\left({}^{*}\omega_n^{[\tau]}\right) \qquad \forall j,\tau \qquad (33) \end{aligned} $$

$$\sum_{r'\neq r}^{|R|}\chi^{r'r[\tau]}E^{r'r[\tau]}t^{r'r[\tau]}+\sum_{r'\neq r}^{|R|}\left[\left({}^{*}\omega^{rr'[\tau]}\right)E^{rr'[\tau]}-E^{rr'[\tau]}t^{rr'[\tau]}-\frac{A^{rr'[\tau]}}{\left({}^{*}\omega^{rr'[\tau]}\right)}\right]=\nabla c^{r[\tau]}-\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_{m}^{jr[\tau]}D_{m}^{j[\tau]}\left(t_{m}^{j[\tau]}+\Delta t_{m}^{j[\tau]}\right)$$
$$-\sum_{i\in\left\{D_{i}^{[\tau]}\right\}^{+}}^{|I|}A_{i}^{r[\tau]}\left[(1-\grave{x}_{i}^{[\tau]})-\frac{\grave{t}_{i}^{[\tau]}}{\left({}^{*}\omega_{i}^{[\tau]}\right)}\right]-\sum_{j=1}^{|J|}\sum_{k\in\left\{D_{k}^{j[\tau]}\right\}^{+}}^{|K(j)|}A_{k}^{jr[\tau]}\left[(1-x_{k}^{j[\tau]})-\frac{t_{k}^{j[\tau]}}{\left({}^{*}\omega_{k}^{j[\tau]}\right)}\right]$$
$$-\sum_{j=1}^{|J|}h^{jr[\tau]}\left[{}^{*}\overline{V^{j[\tau]}}-\left[v^{j[\tau]}+\min\left\{0,\dot{v}^{j[\tau]}\nabla t^{[\tau]}\right\}\right]\right]\nabla t^{[\tau]}-\sum_{i=1}^{|I|}\pi_{i}^{r[\tau]}D_{i}^{[\tau]}\left[(1-\grave{x}_{i}^{[\tau]})\left({}^{*}\omega_{i}^{[\tau]}\right)-\grave{t}_{i}^{[\tau]}\right]$$
$$-\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_{k}^{jr[\tau]}D_{k}^{j[\tau]}\left\{(1-x_{k}^{j[\tau]})\left({}^{*}\omega_{k}^{j[\tau]}\right)-t_{k}^{j[\tau]}-\Delta t_{k}^{j[\tau]}\right\}-\sum_{r'\in\left\{E^{rr'[\tau]}\right\}^{+}}^{|R|}A^{rr'[\tau]}-\sum_{n\in\left\{D_{np}^{[\tau]}\right\}^{+}}^{|N|}\sum_{p\in\left\{D_{np}^{[\tau]}\right\}^{+}}^{|P(n)|}A_{np}^{r[\tau]}\left[\left(1-\grave{x}_{np}^{[\tau]}\right)-\frac{\grave{t}_{n}^{[\tau]}}{\left({}^{*}\omega_{n}^{[\tau]}\right)}-\grave{y}_{np}^{[\tau]}\right]$$
$$-\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\pi_{np}^{j'jr[\tau]}D_{np}^{j'j[\tau]}\left[(1-x'^{[\tau]}_{np}-y'^{[\tau]}_{np})\left({}^{*}\omega_{n}^{[\tau]}\right)-t'^{[\tau]}_{n}-\Delta t_{np}^{[\tau]}\right]\qquad\forall r,\tau\qquad(34)$$

where

$${}^{*}\overline{V^{j[\tau]}}=v^{j[\tau]}+0.5\dot{v}^{j[\tau]}\nabla t^{[\tau]}+\sum_{k=1}^{|K(j)|}\frac{(1-x_{k}^{j[\tau]})}{2}D_{k}^{j[\tau]}\left({}^{*}\omega_{k}^{j[\tau]}\right)+\sum_{m=1}^{|M(j)|}\frac{(1-x_{m}^{j[\tau]})}{2}D_{m}^{j[\tau]}\omega_{m}^{j[\tau]}$$
$$+\sum_{i=1}^{|I|}\frac{(1-x'^{[\tau]}_{i})}{2}g_{i}^{j[\tau]}D_{i}^{[\tau]}\left({}^{*}\omega_{i}^{[\tau]}\right)+\sum_{i=1}^{|I|}\frac{(1-\grave{x}_{i}^{[\tau]})}{2}f_{i}^{j[\tau]}D_{i}^{[\tau]}\left({}^{*}\omega_{i}^{[\tau]}\right)$$
$$+\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\frac{(1-x'^{[\tau]}_{np})}{2}\sum_{j'\neq j}^{|J|}D_{np}^{j'j[\tau]}\left({}^{*}\omega_{n}^{[\tau]}\right)+\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\frac{(1-\grave{x}_{np}^{[\tau]})}{2}\sum_{j'\neq j}^{|J|}D_{np}^{jj'[\tau]}\left({}^{*}\omega_{n}^{[\tau]}\right)\qquad\forall j,\tau\qquad(35)$$

Equation 35 is derived from Eqs. 9 and 33. Equations 7 and 33 indicate that the optimal material storage size is ${}^{*}\overline{V^{j[\tau]}} = 2\,{}^{*}\overline{V^{j[\tau]}}-\left[v^{j[\tau]}+\min\left\{0,\dot{v}^{j[\tau]}\nabla t^{[\tau]}\right\}\right]$. The optimal average level of currency storage, calculated using Eqs. 15 and 34, is

$${}^{*}\overline{C^{r[\tau]}}=c^{r[\tau]}+0.5\dot{c}^{r[\tau]}\nabla t^{[\tau]}+\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}0.5P_{m}^{jr[\tau]}D_{m}^{j[\tau]}(1-x_{m}^{j[\tau]})\omega_{m}^{j[\tau]}+\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}0.5P_{k}^{jr[\tau]}D_{k}^{j[\tau]}(1-x_{k}^{j[\tau]})\left({}^{*}\omega_{k}^{j[\tau]}\right)$$
$$+\sum_{i\in\left\{D_{i}^{[\tau]}\right\}^{+}}^{|I|}0.5A_{i}^{r[\tau]}(1-\grave{x}_{i}^{[\tau]})+\sum_{j=1}^{|J|}\sum_{k\in\left\{D_{k}^{j[\tau]}\right\}^{+}}^{|K(j)|}0.5A_{k}^{jr[\tau]}(1-x_{k}^{j[\tau]})+0.5\sum_{i=1}^{|I|}\pi_{i}^{r[\tau]}D_{i}^{[\tau]}(1-\grave{x}_{i}^{[\tau]})\left({}^{*}\omega_{i}^{[\tau]}\right)$$
$$+\sum_{j=1}^{|J|}h^{jr[\tau]}\left[{}^{*}\overline{V^{j[\tau]}}-\left[v^{j[\tau]}+\min\left\{0,\dot{v}^{j[\tau]}\nabla t^{[\tau]}\right\}\right]\right]\nabla t^{[\tau]}+\sum_{r'\neq r}^{|R|}0.5\chi^{r'r[\tau]}\left({}^{*}\omega^{r'r[\tau]}\right)E^{r'r[\tau]}$$
$$+\sum_{r'\neq r}^{|R|}0.5\left({}^{*}\omega^{rr'[\tau]}\right)E^{rr'[\tau]}+\sum_{r'\in\left\{E^{rr'[\tau]}\right\}^{+}}^{|R|}0.5A^{rr'[\tau]}+\sum_{n\in\left\{D_{np}^{[\tau]}\right\}^{+}}^{|N|}\sum_{p\in\left\{D_{np}^{[\tau]}\right\}^{+}}^{|P(n)|}0.5A_{np}^{r[\tau]}(1-\grave{x}_{np}^{[\tau]})$$
$$+0.5\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\pi_{np}^{j'jr[\tau]}D_{np}^{j'j[\tau]}(1-x'^{[\tau]}_{np})\left({}^{*}\omega_{n}^{[\tau]}\right)\qquad\forall r,\tau\qquad(36)$$

Thus, the optimal currency storage size is ${}^{*}\overline{C^{r[\tau]}}=2\,{}^{*}\overline{C^{r[\tau]}}-\left[c^{r[\tau]}+\min\left\{0,\dot{c}^{r[\tau]}\nabla t^{[\tau]}\right\}\right]$. Then, the optimal value of the objective function, calculated from Eqs. 23–36, is

$$
\begin{aligned}
{}^{*}ATC^{[\tau]}(D_k^{j[\tau]},D_i^{[\tau]},D_n^{jj'[\tau]},E_o^{r[\tau]},v^{j[\tau]},c^{r[\tau]}) &= 2\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\sqrt{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}A_k^{jr[\tau]}\right)\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\Psi_k^{jr[\tau]}\right)D_k^{j[\tau]}} \\
&+\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}P_k^{jr[\tau]}D_k^{j[\tau]} + 2\sum_{i=1}^{|I|}\sqrt{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}A_i^{r[\tau]}\right)\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\Psi_i^{r[\tau]}\right)D_i^{[\tau]}} + \sum_{r=1}^{|R|}\sum_{i=1}^{|I|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}\pi_i^{r[\tau]}D_i^{[\tau]} \\
&+2\sum_{n=1}^{|N|}\sqrt{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}\sum_{p=1}^{|P(n)|}A_{np}^{r[\tau]}\right)\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{p=1}^{|P(n)|}\Psi_{np}^{jj'r[\tau]}D_{np}^{jj'[\tau]}\right)} + 2\sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\tilde{\chi}^{r1[\tau]}\sqrt{\mu^{r[\tau]}A^{rr'[\tau]}\Psi^{rr'[\tau]}E^{rr'[\tau]}} \\
&+\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\left[\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}\pi_{np}^{jj'r[\tau]} + \sum_{r'\neq r}^{|R|}\left\{\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]} - \tilde{\chi}^{r'1[\tau]}\chi^{rr'[\tau]}\mu^{r'[\tau]}\right\}P_{jj'}^{rr'[\tau]}\right]D_{np}^{jj'[\tau]} \\
&+\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\tilde{\chi}^{r1[\tau]}\left(\mu^{r[\tau]}h^{jr[\tau]}+\gamma^{jr[\tau]}\right)0.5\left(v^{j[\tau]}+v^{j[\tau+1]}\right) + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}b^{jr[\tau]}\left(\max\left\{v^{j[\tau]},v^{j[\tau+1]}\right\}\right) \\
&+0.5\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}h^{jr[\tau]}\left(\nabla t^{[\tau]}\right)\left[\max\left\{v^{j[\tau]},v^{j[\tau+1]}\right\}-\min\left\{v^{j[\tau]},v^{j[\tau+1]}\right\}\right] + 0.5\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}\sum_{i\in\left\{D_i^{[\tau]}\right\}^+}^{|I|}(1-\grave{x}_i^{[\tau]})A_i^{r[\tau]} \\
&+0.5\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}\sum_{j=1}^{|J|}\sum_{k\in\left\{D_k^{j[\tau]}\right\}^+}^{|K(j)|}(1-x_k^{j[\tau]})A_k^{jr[\tau]} + 0.5\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}\sum_{n\in\left\{D_n^{[\tau]}\right\}^+}^{|N|}\sum_{p=1}^{|P(n)|}(1-\grave{x}_{np}^{[\tau]})A_{np}^{r[\tau]} + 0.5\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}\sum_{r'\in\left\{E^{rr'[\tau]}\right\}^+}^{|R|}A^{rr'[\tau]} \\
&+\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}c^{r[\tau]} - 0.5\nabla t^{[\tau]}\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}\sum_{o\neq\bar{o}}^{|O|}E_o^{r[\tau]} + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\tilde{\chi}^{r1[\tau]}\left(\frac{\eta^{r[\tau]}P_m^{jr[\tau]}}{2}+\theta^{jr[\tau]}\right)(1-x_m^{j[\tau]})D_m^{j[\tau]}\omega_m^{j[\tau]} \\
&-\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}P_m^{jr[\tau]}D_m^{j[\tau]} \quad \forall\tau \qquad (37)
\end{aligned}
$$

The second-level optimization problem entails

P2: Minimizing the objective function in Eqs. 19 and 37, under the constraints given in Eqs. 5, 14, and 21 with respect to the design variables $D_k^{j[\tau]}, D_i^{[\tau]}, D_{np}^{jj'[\tau]}, E_o^{r[\tau]}$, $v^{j[\tau]}$, and $c^{j[\tau]}$.

The second-level problem should be solved first to obtain the optimal values for the average flow rates of the materials and currencies. Then, using Eqs. 23–32, the optimal values for the cycle times and process capacities can be calculated. The optimal capacity of the material storage units is calculated using ${}^{*}\overline{V^{j[\tau]}} = 2{}^{*}\overline{V^{j[\tau]}} - \left[v^{j[\tau]} + \min\left\{0, \dot{v}^{j[\tau]}\nabla t^{[\tau]}\right\}\right]$ and Eq. 35. Finally, the startup times of the material and currency flows are calculated using Eqs. 33 and 34 under the condition of minimizing $\nabla v^{j[\tau]}$ and $\nabla c^{r[\tau]}$. Positive values of $\nabla v^{j[\tau]}$ and $\nabla c^{r[\tau]}$ should be supplied from the third parties, which cause additional costs.

The main difference between the optimal solutions to the single-period model with infinite horizon[5,10] and the multiperiod model with finite horizon is the presence of the terms that include $\nabla t^{[\tau]}$ in Eqs. 27 and 28. The terms that include $\nabla t^{[\tau]}$ originate from the opportunity cost associated with the currency inventory $\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\eta^{r[\tau]}\overline{C^{r[\tau]}}$ in Eq. 20. Note that $\nabla t^{[\tau]}$ is a model parameter and not a system parameter. Optimal lot-sizes decrease and optimal cost increases as $\nabla t^{[\tau]}$ increases, although these changes are small, as can be seen at Figure 5. This demonstrates that the modeler's perspective of the system can influence the optimal solution. The solutions for the single-period model correspond to the case in which $\nabla t^{[\tau]} = 0$.

Multiperiod Planning Model

The second-level optimization problem constitutes a multiperiod planning model. We introduce a set of constraints in addition to Eqs. 5, 14, and 21. The capacities of the processes have both lower and upper limits. For example, the lower limit for the production process i, $B_i^{[\tau]} \equiv D_i^{[\tau]}\omega_i^{[\tau]} \geq \underline{\underline{B_i^{[\tau]}}}$, and Eq. 24 gives

$$
D_i^{[\tau]} \geq \left(\underline{\underline{B_i^{[\tau]}}}\right)^2 \frac{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\Psi_i^{r[\tau]}\right)}{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}A_i^{r[\tau]}\right)} \quad \forall i,\tau \qquad (38)
$$

The upper limit for the transportation process n, $B_n^{[\tau]} = \left(\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{p=1}^{|P(n)|}D_{np}^{jj'[\tau]}\right)\omega_n^{[\tau]} \leq \overline{\overline{B_n^{[\tau]}}}$, and Eq. 25 gives

$$
\begin{aligned}
&\sqrt{\left(\sum_{r=1}^{|R|}\sum_{p=1}^{|P(n)|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}A_{np}^{r[\tau]}\right)}\left(\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{p=1}^{|P(n)|}D_{np}^{jj'[\tau]}\right) \\
&\leq \overline{\overline{B_n^{[\tau]}}}\sqrt{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{p=1}^{|P(n)|}\Psi_{np}^{jj'r[\tau]}D_{np}^{jj'[\tau]}\right)} \quad \forall n,\tau \qquad (39)
\end{aligned}
$$

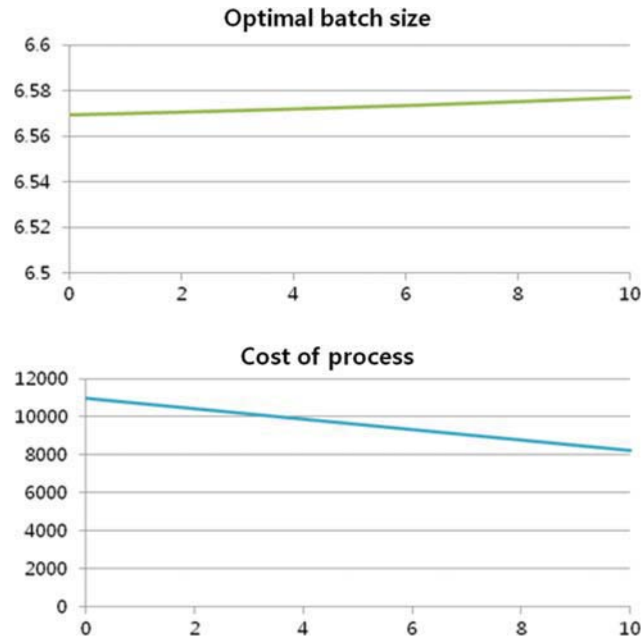

Figure 5. Effect of time period interval.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

The material storage capacities can be limited, that is, $^*\overline{\boldsymbol{V}^{j[\tau]}} \le \boldsymbol{V}^{j}_{\max}$. Eqs. 24–26 and 35, and $^*\overline{\boldsymbol{V}^{j[\tau]}} = \mathbf{2}^*\overline{\boldsymbol{V}^{j[\tau]}} - \left[\boldsymbol{v}^{j[\tau]} + \min\left\{\mathbf{0}, \dot{\boldsymbol{v}}^{j[\tau]}\nabla \boldsymbol{t}^{[\tau]}\right\}\right]$ give

$$
\begin{aligned}
V^{j}_{\max} \ge{}& \max\left\{v^{j[\tau]}, v^{j[\tau+1]}\right\} \\
&+\sum_{k=1}^{|K(j)|}(1-x_k^{j[\tau]})\sqrt{\frac{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}A_k^{jr[\tau]}\right)D_k^{j[\tau]}}{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\Psi_k^{jr[\tau]}\right)}} \\
&+\sum_{i=1}^{|I|}\left[(1-x'^{[\tau]}_i)g_i^{j[\tau]}+(1-\grave{x}_i^{[\tau]})f_i^{j[\tau]}\right]\sqrt{\frac{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}A_i^{r[\tau]}\right)D_i^{[\tau]}}{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\Psi_i^{r[\tau]}\right)}} \\
&+\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\frac{(1-x'^{[\tau]}_{np})}{2}\sum_{j'\neq j}^{|J|}D_{np}^{j'j[\tau]}\left({}^*\omega_n^{[\tau]}\right)+\sum_{n=1}^{|N|}\sum_{p=1}^{|P(n)|}\frac{(1-\grave{x}_{np}^{[\tau]})}{2} \\
&\times\sum_{j'\neq j}^{|J|}D_{np}^{jj'[\tau]}\left({}^*\omega_n^{[\tau]}\right)+\sum_{m=1}^{|M(j)|}(1-x_m^{j[\tau]})D_m^{j[\tau]}\omega_m^{j[\tau]} \qquad \forall j,\tau \qquad (40)
\end{aligned}
$$

Note that $D_{np}^{j'j[\tau]}({}^*\omega_n^{[\tau]})$ and $D_{np}^{jj'[\tau]}({}^*\omega_n^{[\tau]})$, where ${}^*\omega_n^{[\tau]}$ is given by Eq. 25, are nonseparable. A subset of the inequalities defined in Eq. 40, which is computationally easy to apply as well as sufficient for solving the problem, is achieved by replacing $D_{np}^{j'j[\tau]}({}^*\omega_n^{[\tau]})$ and $D_{np}^{jj'[\tau]}({}^*\omega_n^{[\tau]})$ with the separable upper bounds

$$
D_{np}^{jj'[\tau]}\left({}^*\omega_n^{[\tau]}\right) \le \sqrt{\frac{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}\sum_{p=1}^{|P(n)|}A_{np}^{r[\tau]}\right)D_{np}^{j'j[\tau]}}{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\Psi_{np}^{j'jr[\tau]}\right)}}
$$

and

$$
D_{np}^{jj'[\tau]}\left({}^*\omega_n^{[\tau]}\right) \le \sqrt{\frac{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\mu^{r[\tau]}\sum_{p=1}^{|P(n)|}A_{np}^{r[\tau]}\right)D_{np}^{jj'[\tau]}}{\left(\sum_{r=1}^{|R|}\tilde{\chi}^{r1[\tau]}\Psi_{np}^{jj'r[\tau]}\right)}}
$$

The equalities in the upper bounds of $D_{np}^{j'j[\tau]}({}^*\omega_n^{[\tau]})$ and $D_{np}^{jj'[\tau]}({}^*\omega_n^{[\tau]})$ hold when the transportation process n carries a single material in a single parcel. Note that the square root terms in Eq. 40 are similar to the square root terms in the objective function Eq. 37, and they only differ in their coefficients. Therefore, the optimal solution, in the absence of the constraint given by Eq. 40, also tends to reduce the maximum inventory levels on the right side of the inequality in Eq. 40.

The second-level optimization problem is a nonconvex mixed integer nonlinear programming formulation with separable concave terms (square roots) in the objective function given in Eq. 37 and in the constraints given in Eqs. 21, 39, and 40, and with nonseparable terms in the constraints given in Eqs. 21 and 40, $D_{np}^{jj'[\tau]}({}^*\omega_n^{[\tau]})$. $A_i^{r[\tau]}$, $A_k^{jr[\tau]}$, $A_{np}^{r[\tau]}$, and $A^{rr'[\tau]}$ should be zero if their corresponding average flow rates approach zero at the optimum conditions. To address this issue, the objective function in Eq. 37 should include binary variables that exclude the setup costs, the average flow rates of which approach zero. For example, $\sum_{r'\in\left\{E^{rr'[\tau]}\right\}^+}^{|R|}A^{rr'[\tau]} = \sum_{r'=1}^{|R|}A^{rr'[\tau]}z^{rr'[\tau]}$, where the binary variable $z^{rr'[\tau]} = 1$ if $E^{rr'[\tau]} > 0$ and $z^{rr'[\tau]} = 0$ otherwise. Note that $\max\left\{v^{j[\tau]}, v^{j[\tau+1]}\right\}$ and $\min\left\{v^{j[\tau]}, v^{j[\tau+1]}\right\}$ are replaced with the linear constraints $\max\left\{v^{j[\tau]}, v^{j[\tau+1]}\right\} \ge v^{j[\tau]}$ and $\min\left\{v^{j[\tau]}, v^{j[\tau+1]}\right\} \le v^{j[\tau]}$, respectively. Equation 11 is reformulated as follows.

$$
\begin{aligned}
&\chi^{r1[\tau]}\overline{\overline{E^{rr'[\tau]}}} - \chi^{r'1[\tau]}\overline{\overline{E^{r'r[\tau]}}} \le \chi^{r1[\tau]}(\mathrm{big}M)z^{rr'[\tau]} \\
&\chi^{r1[\tau]}\overline{\overline{E^{rr'[\tau]}}} - \chi^{r'1[\tau]}\overline{\overline{E^{r'r[\tau]}}} \ge -\chi^{r'1[\tau]}(\mathrm{big}M)\left(1 - z^{rr'[\tau]}\right) \\
&\chi^{r1[\tau]}\overline{\overline{E^{rr'[\tau]}}} - \chi^{r'1[\tau]}\overline{\overline{E^{r'r[\tau]}}} = \chi^{r1[\tau]}E^{rr'[\tau]} - \chi^{r'1[\tau]}E^{r'r[\tau]} \\
&E^{rr'[\tau]} \le (\mathrm{big}M)z^{rr'[\tau]} \\
&z^{rr'[\tau]} + z^{r'r[\tau]} = 1
\end{aligned}
$$

Note that the mathematical structure of the multiperiod planning model should be relatively easy to compute because the model commonly has a huge number of equations and variables. Because there is no easy way to deal with the nonseparable terms in an equality constraint, we remove the nonseparable term in Eq. 21, $\sum_{n=1}^{|N|}a_n^{r[\tau]}D_n^{[\tau]}\omega_n^{[\tau]}$, which represents annualized capital investment cost for transportation processes. All terms in the modified multiperiod planning model are linear or nonlinear but separable. Separable terms can be linearized using separable programming techniques,[4] and, therefore, the model falls in the category of mixed integer linear programs. The unique feature of the multiperiod planning model is the square roots of the average flow rates in the objective function. This objective function originated from the setup and inventory holding costs of the materials and currencies, which depend on the operation frequency. These square root terms increase the model accuracy but also increase the computational burden. The square root

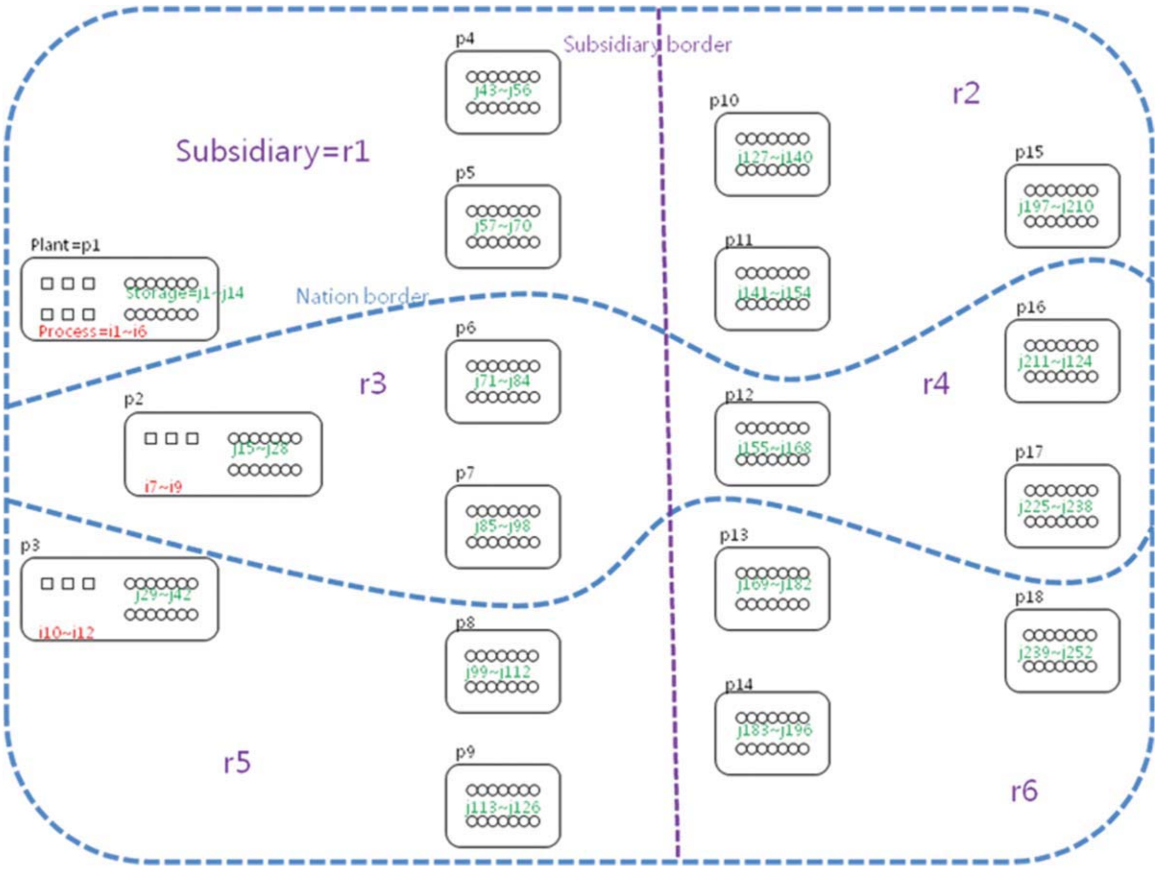

Figure 6. Example supply chain layout of multinational corporation.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

terms can be approximated by a piecewise linear function composed of at least 5 segments of straight lines that yield a deviation of 1 %. Therefore, one square root term produces at least 5 Specially Ordered Sets type 2 (SOS2) variables[4] and 2 equations. The multiperiod planning model usually has many square root terms and the increase in model size is substantial. The following section will explain how to solve a multiperiod planning model characterized by a real size problem.

Discussion of the Computational Results

Figure 6 illustrates an example production inventory and distribution system that this study was designed to handle. This example is composed of 18 plants (rounded rectangles), 14 materials (circles), and 12 production processes (squares). The first three plants possess production batch processes $i1$–$i12$ and material storage units $j1$–$j42$. The other plants include only material storage units $j43$–$j252$. Plant $p4$–$p9$ correspond to distribution centers, and the other plants correspond to terminals. The system purchases four raw materials and manufactures all other 10 materials at plants $p1$–$p3$ (production plants). The materials are then transported to all other plants. We assume that there are no transportation deliveries from plants $p4$–$p18$ (distribution centers and terminals) to plants $p1$–$p3$ (production plants) and no multiproduct deliveries from plants $p1$–$p9$ (production plants and distribution centers) to $p10$–$p18$ (terminals). The number of possible transportation routes is 24 for a multiproduct delivery and 1112 for a single product delivery. The supply chain network is owned by six subsidiaries $r1$–$r6$, as indicated by the dotted lines in Figure 6. Two horizontally curved dotted lines represent the borders between nations, and there are three nations. Each nation has two subsidiaries: the first nation has $r1/r2$, the second nation has $r3/r4$, and the third nation has $r5/r6$. Each nation uses a single currency, and each subsidiary uses the currency claimed by its nation. Therefore, $\chi^{12[\tau]} = \chi^{34[\tau]} = \chi^{56[\tau]} = 1$. The currency of the first nation is numeraire. We assume that there is no arbitrage in the currency exchange, that is, $\chi^{rr'[\tau]} = \frac{1}{\chi^{r'r[\tau]}}$. In reality, $\chi^{rr'[\tau]} \neq \frac{1}{\chi^{r'r[\tau]}}$, for example, the purchase price of 1 US dollar in Korea is 1179 Won/\$, and the selling price of 1 US dollar in Korea is 1135 Won/\$ (3.7% deviation). The currency exchange rates are set to $\chi^{13[\tau]} = \chi^{35[\tau]} = 10$. The corporate income taxes are set to $\xi^{1[\tau]} = \xi^{2[\tau]} = 0.35$, $\xi^{3[\tau]} = \xi^{4[\tau]} = 0.28$, and $\xi^{5[\tau]} = \xi^{6[\tau]} = 0.07$. The interest rates are set to $\eta^{1[\tau]} = \eta^{2[\tau]} = 0.01$, $\eta^{3[\tau]} = \eta^{4[\tau]} = 0.05$, and $\eta^{5[\tau]} = \eta^{6[\tau]} = 0.11$. We assume that all SOTFs are zero for convenience. The time period interval $\triangledown t^{[\tau]}$ is set to 1 year. We would like to optimize the sizes for production/transportation processes and material storage, and optimize the multiperiod material/currency inventories and average material/currency flow rates through the routes.

The multiperiod planning model to determine the average material/currency flow rates and the multiperiod material/

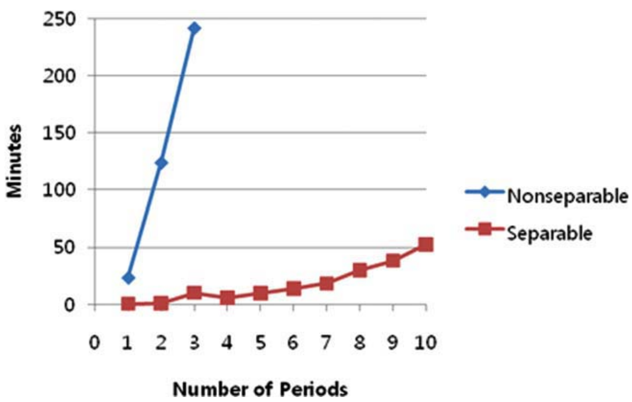

Figure 7. CPU time with respect to number of periods.

currency inventories can be formulated as an MINLP with (P2) and additional constraints given in Eqs. 38–40. This MINLP model has 1175 binary variables, 3576 continuous variables, and 3429 equations with 336 nonseparable terms per each time period. No commercialized global optimizer exist that can solve a problem with large binary numbers in MINLP. GAMS/DICOPT 2x-c can calculate a local optimum point very quickly only if a suitable initial guess is given. We select separable programming technique for nonseparable function[22] as is summarized in Supporting Information Appendix B posted in http://myweb.pknu.ac.kr/gbyi/. This technique transforms nonseparable MINLP to large-scale MILP and thus we can take the advantage of powerful MILP commercial solvers. MILP also gives a near global optimum as far as branch and bound procedure is fully developed. Separable programming technique is based on the fact that any nonlinear function can be approximated by sufficient number of pieces of linear functions. Our problem has nonseparable terms of the type $\frac{x}{\sqrt{y}}$. To achieve for the function evaluation error produced by the linearization procedure to be less than 1%, we divide the x and y of the nonseparable terms into 60 equidistant grid points. The resulting MILP model has 1175 binary variables, 1,326,108 continuous variables (1550 SOS2 sets), and 49,062 equations per each time period. The MILP model is solved by GAMS/CPLEX 12.2.0, calculated using an Intel(R) Core(TM)2 Quad CPU Q9500 @2.83 GHz with 8 GB RAM. The optimality gap is set to 4% because the first feasible solution is obtained within 4% gap. The solution to the MILP model is a near global optimum but still distant from a global optimum because of loose optimality gap. Tightening optimality gap is very costly and instead, we inputted the solution to the MILP model as an initial guess of MINLP model by using GAMS/DICOPT 2x-c. It took 23.2 min for one period model

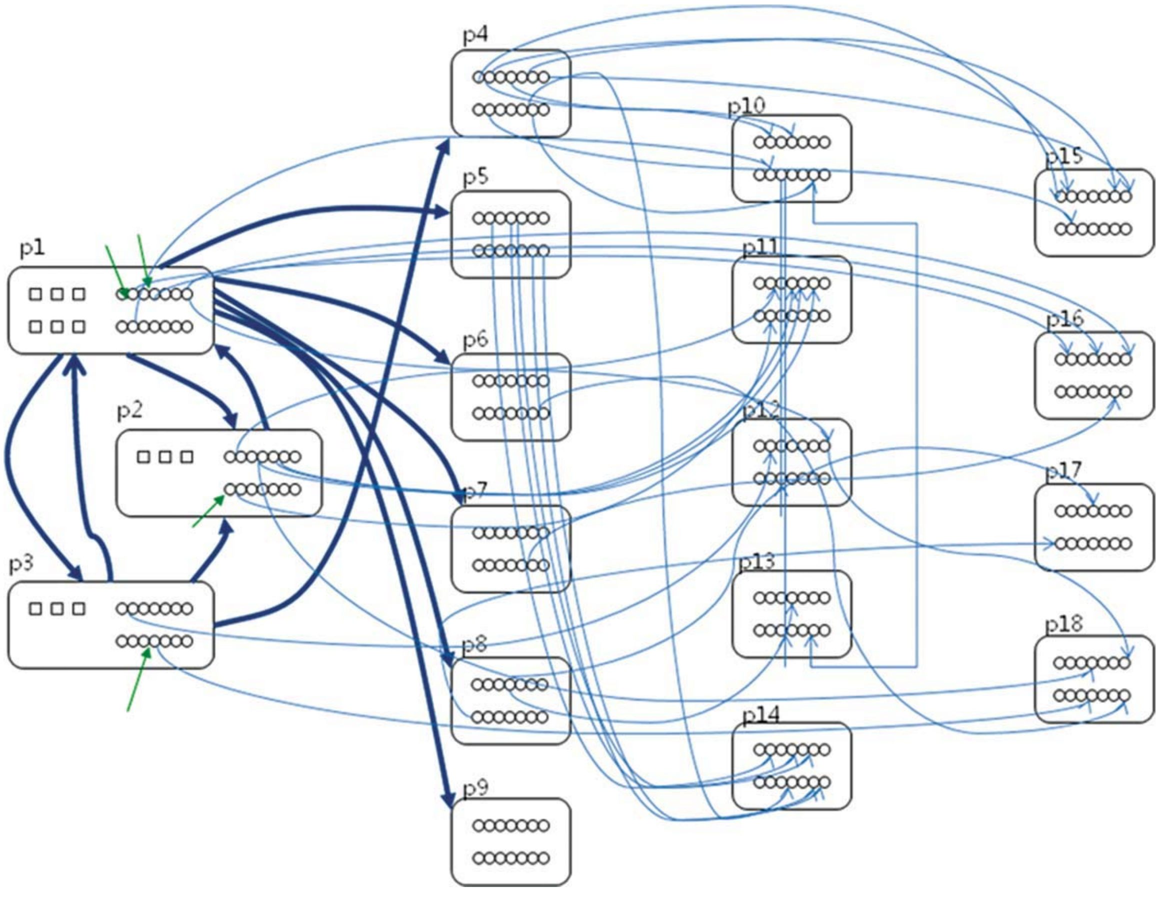

Figure 8. Optimized supply chain network of the example.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

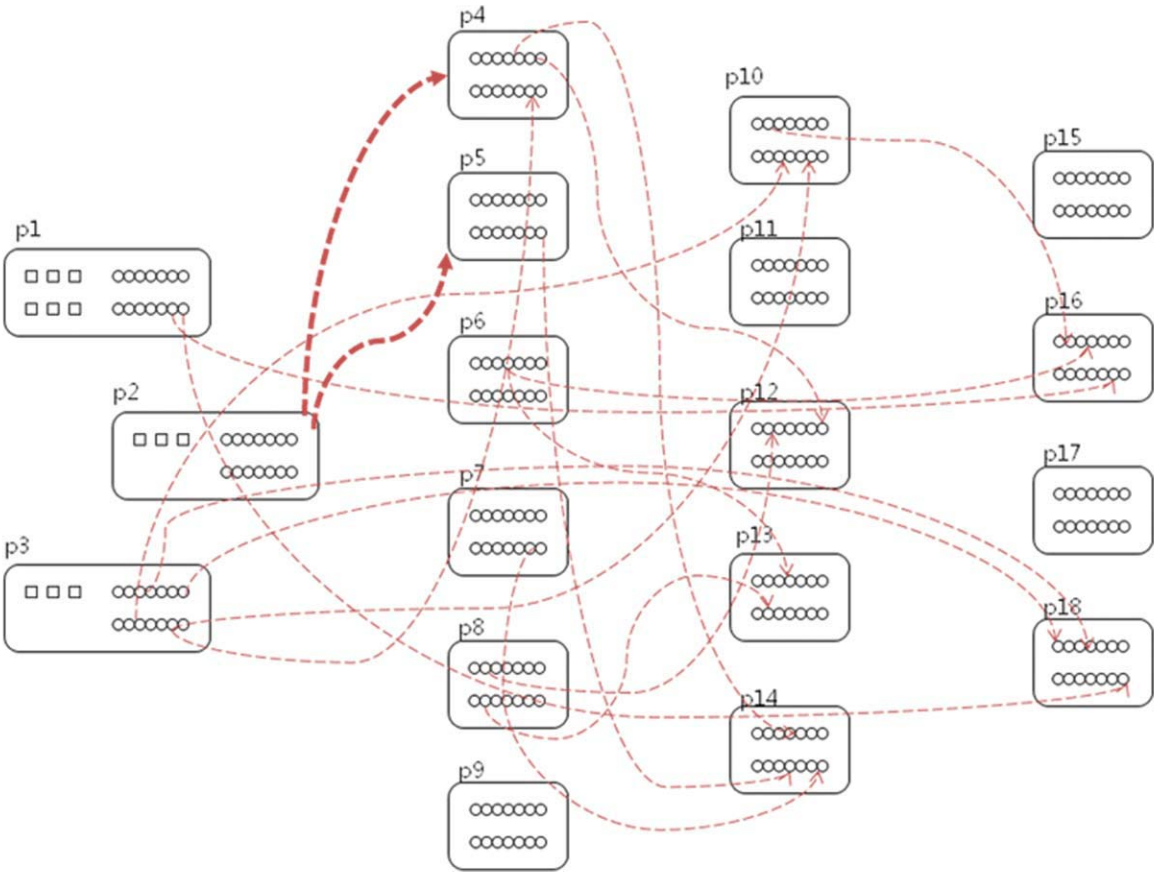

Figure 9. Optimized supply chain network of the example without corporate income taxes.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

and 123.8 min for two period model. We could not obtain the solution for three period model by using equidistant grid points because memory requirement exceeded 8 GB. Model size can be reduced by using nonequidistant grid points. $\frac{x}{\sqrt{y}}$ becomes very stiff as y approaches to zero. We position more grid points where the function becomes stiff in such a way that the interval between grid points increases linearly as y increases. Then, we can reduce the grid points of x and y by 25 and 50 and thus the number of continuous variables is reduced to 64% and the number of equations is reduced to 31% without sacrificing the function evaluation error. Figure 7 shows the computation times for solutions to the MILP model named "Nonseparable" over the time periods 1–3. If we cannot remove the annualized capital investment cost for transportation processes in Eq. 21, $\sum_{n=1}^{|N|} a_n^{r[\tau]} D_n^{[\tau]} \omega_n^{[\tau]}$, we have to stick to this "Nonseparable" model in spite of heavy computational burden. If we can remove this nonseparable term in equality constraint, we can develop a more efficient model named "Separable."

The nonseparable terms in Eq. 40 are replaced with separable upper bound terms, and separable terms (square roots) are linearized by introducing SOS2 variables with 5 equal intervals. The function evaluation error produced by the linearization procedure is less than 0.5%, and the error introduced by Eq. 40, in replacing nonseparable terms with separable terms, is less than 16%. The linearized MILP model includes 1175 binary variables, 14,258 continuous variables (1526 SOS2 sets), and 7992 equations per time period. The feasible zone of the MILP model encompasses a subset of the feasible zone of the MINLP model because the nonseparable terms in the inequality constraint given in Eq. 40 are replaced with their upper bounds. Obviously, a solution exists that is better than the optimal solution to the MILP model. It is a good strategy to use the optimal solution to the MILP model as an initial guess for the MINLP model when using GAMS/DICOPT 2x-c. Additional computational time for the MINLP model was less than 4 min. The solution to the MINLP model improved the solution to the MILP model about 1.1%. Figure 7 shows the computation times for solutions to the MILP + MINLP model named "Separable" over the time periods 1–10. The optimality gap is set to 4% because the first feasible solution is obtained within 4% gap. Reducing optimality gap to 3% consumes about 17 times more CPU time but the objective value of the MILP + MINLP is reduced by only 0.05%. The optimal solutions computed with the "Separable" model are only 0.5% higher than those computed with the "Nonseparable" model. This shows that the multiperiod planning model based on BSN developed in this study can be applied toward a real size problem. Recent work introduces a branch-and-refine algorithm for the rigorous global optimization of MINLP with square root terms.[23]

Figure 8 shows an optimized supply chain network with a single finite period solution result. Ten multiproduct transportation routes are selected from 24 candidate routes. Thirty-six single product transportation routes are selected from 1112 candidate routes. Bold lines represent multiproduct delivery, and thin lines represent single product delivery. Figure 9 shows the case in which no corporate income taxes were required to be paid. The dotted lines represent newly added routes. Eighteen of the 46 routes in Figure 8 are replaced. Eleven of the 46 routes in Figure 8 are replaced in the case in which no interest rates were required to be paid. Fourteen of the 46 routes in Figure 8 are replaced in the case in which $\chi^{13[\tau]} = \chi^{35[\tau]} = 8$ (20% reduction). The fact that 20–40% of the transportation routes changed reveals that financial factors, such as corporate income taxes, interest rates, and exchange rates, must not be ignored during supply chain optimization.

Conclusions

A multiperiod planning model based on a BSN was developed and tested using a real size problem. The multiperiod planning model based on a BSN has many advantages over existing planning models: it can easily account for the operation frequency dependent costs in separable square root terms; it provides optimal operation frequencies with no additional computational requirement; it can be easily applied to MNCs in which international financial factors, such as exchange rates, taxes, and interest rates, are important; and it easily incorporates uncertainties.[7,8] However, it also has some disadvantages relative to existing models. Because the time period intervals must be much larger than the operation cycle times and any other timing parameters, the timescale on which this multiperiod planning model, based on a BSN, can be applied are restricted to strategic or tactical planning timescales. This may be corrected by introducing different time frames in the time period and is left as an element of future work.

In this study, we introduced transportation processes into the BSN that carry multiple products at once. Transportation processes that carry multiple products generate cost data that cannot be separated product by product. Such cost data requires special treatment during modeling.

One of the great advantages of the BSN arises from the fact that the optimal solution to the BSN system takes the form of a set of analytical lot-sizing equations. The lot-sizing equations derived from multiperiod formulations with finite horizon are different from those of single-period formulations with infinite horizon. The difference between single-period and multiperiod solutions rests in the inclusion of a time period interval in the solution. The time period intervals should be chosen so that variations in the system parameters are negligible within a time period. The time period interval is a modeling parameter that the designer can choose subjectively. The fact that a modeling parameter is involved in the optimal solution indicates that the model designer's personal experience is important for achieving the best performance.

We derived equations with nonseparable terms that compute the currency flows. We also solved the planning model with nonseparable terms. A special separable programming technique for nonseparable functions is applied to obtain the global optimum of the nonconvex MINLP. The performance was limited but very promising in the future. The nonseparable terms in inequality constraints are replaced with separable upper bound terms. This simplified planning model showed good computational performance although its application is restricted to the case that the annualized capital investment cost for transportation processes is not the major concern.

Acknowledgments

This work was supported by the Pukyong National University Research Abroad Fund in 2009(PS-2009-026).

Notations

Subscript

i = production process index
k = index of raw material vendors
n = index of transportation process
o = index of stockholders
p = index of parcels
$\bar{o}$ = index of corporate income tax

Superscript

j = storage index
r = index of currency in subsidiary
$[\tau]$ = time period index, $0 \leq \tau \leq T$

$a_k^{jr[\tau]}$ = annualized capital cost of raw material purchasing facility, currency/L/year, parameter
$a_i^{r[\tau]}$ = annualized capital cost of production process i paid by currency r, currency/L/year, parameter
$a_n^{r[\tau]}$ = annualized capital cost of transportation process n paid by currency r, currency/L/year, parameter
$b^{jr[\tau]}$ = annualized capital cost of storage facility j paid by currency r, currency/L/year, parameter
$A_k^{jr[\tau]}$ = setup cost of feedstock materials, currency/batch, parameter
$A_i^{r[\tau]}$ = setup cost of noncontinuous units, currency/batch, parameter
$A_{np}^{r[\tau]}$ = setup cost of parcel p in transportation process n, currency/batch, parameter
$A^{rr'[\tau]}$ = setup cost of currency transfer, currency/transaction, parameter
big$M$ = some large number, parameter
$B_k^{j[\tau]}$ = raw material order size, L/batch, variable
$B_i^{[\tau]}$ = production process batch size, L/batch, variable
$\overline{\overline{B_i^{[\tau]}}}$ = maximum of $B_i^{[\tau]}$, L/batch, parameter
$B_m^{j[\tau]}$ = final product delivery size, L/batch, parameter
$B_n^{[\tau]}$ = transportation process batch size, L/batch, variable
$\overline{\overline{B_n^{[\tau]}}}$ = maximum of $B_n^{[\tau]}$, L/batch, parameter
$B_{np}^{[\tau]}$ = batch size of parcel p in transportation process n, L/batch, variable
$c^{r[\tau]}$ = multiperiod currency level at time period τ, currency, decision variable
$\dot{c}^{r[\tau]}$ = currency imbalance rate at time period τ, currency/year, variable
$\nabla c^{r[\tau]}$ = $\equiv C^{r[\tau]}(0) - c^{r[\tau]}$, L, variable
$C^{r[\tau]}(0)$ = initial cash inventory of currency r, L, variable
$C^{r[\tau]}(t)$ = cash inventory of currency r at present time t, L, variable
$\overline{C^{r[\tau]}}$ = average level of currency inventory, L, variable
$\overline{\overline{C^{r[\tau]}}}$ = upper level of currency inventory, L, variable
$\underline{\underline{C^{r[\tau]}}}$ = lower level of currency inventory, L, variable
$D_k^{j[\tau]}$ = average material flow of raw material supply, L/year, decision variable
$\left\{D_k^{j[\tau]}\right\}^+$ = The set of index k that has positive value of $D_k^{j[\tau]}$, variable
$D_m^{j[\tau]}$ = average material flow of customer demand, L/year, parameter
$D_i^{[\tau]}$ = average material flow through noncontinuous units, L/year, decision variable
$\left\{D_i^{[\tau]}\right\}^+$ = the set of index i that has positive value of $D_i^{[\tau]}$, variable
$D_{np}^{jj'[\tau]}$ = average material flow rate from storage j to storage j' via parcel p in transportation process n, L/year, decision variable

$\left\{D_{np}^{jj'[\tau]}\right\}^+$ = the set of index n or p that has positive value of $D_n^{jj'[\tau]}$, variable
$E_o^{r[\tau]}$ = average currency flow rate of dividend to stockholders o, currency/year, decision variable
$E_{\bar{o}}^{r[\tau]}$ = average currency flow rate of corporate income tax, currency/year, variable
$E^{rr'[\tau]}$ = average currency flow rate of currency transfer from r to r', currency/year, variable
$\overline{\overline{E^{rr'[\tau]}}}$ = $\equiv \sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N|} P_{j'j}^{rr'[\tau]} D_n^{j'j[\tau]}$, currency/year, variable
$\left\{E^{rr'[\tau]}\right\}^+$ = The set of index r, r' that has positive value of $E^{rr'[\tau]}$, variable
$F_i^{[\tau]}(t)$ = periodic square wave flow, L/year, variable
$f_i^{j[\tau]}$ = feedstock composition of production unit i, parameter
$g_i^{j[\tau]}$ = product yield of production unit i, parameter
$h^{jr[\tau]}$ = annual inventory operating cost, currency/L/year, parameter
$H^{jr[\tau]}$ = $\equiv h^{jr[\tau]} + \gamma^{jr[\tau]}$, annual inventory holding cost, currency/L/year, parameter
I = batch production process set, parameter
$I(r)$ = batch production process subset owned by the subsidiary that uses currency r, parameter
J = material storage set, parameter
$J(r)$ = material storage subset owned by the subsidiary that uses currency r, parameter
$K(j)$ = raw material supplier set for storage j, parameter
$M(j)$ = consumer set for storage j, parameter
$P(n)$ = parcel set for transportation process n, parameter
$P_k^{jr[\tau]}$ = price of raw material j from supplier k paid by currency r, currency/L, parameter
$P_m^{jr[\tau]}$ = sales price of finished products to customer m paid by currency r, currency/L, parameter
$P_{j'j}^{rr'[\tau]}$ = transfer price represented by currency r and transferred from currency storage $r \in R(j)$ to currency storage $r' \in R(j')$ for the material transported from storage $j' \in J(r')$ to storage $j \in J(r)$, currency/L, parameter
PSW = the first type of periodic square wave function defined by Eq. 1, variable
PSW′ = the second type of periodic square wave function defined by Eq. 2, variable
$\overline{\text{PSW}}$ = average level of the first type of periodic square wave function defined in Table 1 of Ref. 10, variable
$\overline{\text{PSW}'}$ = average level of the second type of periodic square wave function defined in Table 1 of Ref. 10, variable
$\overline{\overline{\text{PSW}}}$ = upper bound of the first type of periodic square wave function defined in Table 1 of Ref. 10, variable
$\overline{\overline{\text{PSW}'}}$ = upper bound of the second type of periodic square wave function defined in Table 1 of Ref. 10, variable
$\underline{\underline{\text{PSW}}}$ = lower bound of the first type of periodic square wave function defined in Table 1 of Ref. 10, variable
$\underline{\underline{\text{PSW}'}}$ = lower bound of the second type of periodic square wave function defined in Table 1 of Ref. 10, variable
R = currency set, parameter
$t_m^{j[\tau]}$ = startup time of customer demand, year, parameter
$\grave{t}_i^{[\tau]}$ = startup time of feedstock feeding to batch production process i, year, decision variable
$t_i'^{[\tau]})$ = startup time of product discharging from batch production process i, year, variable
$t_k^{j[\tau]}$ = startup time of raw material purchasing, year, decision variable
$\grave{t}_n^{[\tau]}$ = startup time of loading of transportation process n, year, decision variable
$t_n'^{[\tau]}$ = startup time of unloading of transportation process n, year, variable
$\grave{t}_{np}^{[\tau]}$ = startup time of loading of parcel p in transportation process n, year, variable
$t_{np}'^{[\tau]}$ = startup time of unloading of parcel p in transportation process n, year, variable
$t_o^{r[\tau]}$ = startup time of dividend to stockholders, year, parameter
$t^{rr'[\tau]}$ = startup time of currency transfer, year, decision variable
$\Delta t_i^{[\tau]}(.)$ = the time lag of batch production process i defined by Eq. 3, year, variable
Δt_k^j = disbursement drifting time of account payables, year, parameter
Δt_m^j = collection drifting time of account receivables, year, parameter
$\Delta t_{np}^{[\tau]}$ = the time lag of parcel p in transportation process n defined by Eq. 4, year, parameter
$\nabla t^{[\tau]}$ = the length of time period τ, year, parameter
$V_{\max}^j$ = maximum material inventory limit, L, parameter
$\overline{\overline{V^{j[\tau]}}}$ = upper bound of material inventory hold-up, L, variable
$\underline{\underline{V^{j[\tau]}}}$ = lower bound of material inventory hold-up, L, variable
$V^{j[\tau]}(t)$ = material inventory hold-up, L, variable,
$V^{j[\tau]}(0)$ = initial material inventory hold-up, L, parameter
$\overline{V^{j[\tau]}}$ = time averaged material inventory hold-up, L, variable
$v^{j[\tau]}$ = multiperiod material inventory at time period τ, L, decision variable
$\dot{v}^{j[\tau]}$ = material imbalance at time period τ, L, variable
$\nabla v^{j[\tau]}$ = $\equiv V^{r[\tau]}(0) - v^{r[\tau]}$, L, variable
$x_k^{j[\tau]}$ = storage operation time fraction of purchasing raw materials, parameter
$\grave{x}_i^{[\tau]}$ = storage operation time fraction of feeding to batch production process i, parameter
$x_i'^{[\tau]}$ = storage operation time fraction of discharging from batch production process i, parameter
$x_m^{j[\tau]}$ = storage operation time fraction of customer demand, parameter
$\grave{x}_{np}^{[\tau]}$ = storage operation time fraction of loading to transportation process n, parameter
$x_{np}'^{[\tau]}$ = storage operation time fraction of unloading from transportation process n, parameter
$\grave{y}_{np}^{[\tau]}$ = defined by Eq. 4, parameter
$x_{np}'^{[\tau]}$ = defined by Eq. 4, parameter
$z_k^{jr[\tau]}$ = binary variable, = 1 if $D_k^{jr[\tau]} > 0$, = 0 otherwise, variable
$z_i^{r[\tau]}$ = binary variable, = 1 if $D_i^{r[\tau]} > 0$, = 0 otherwise, variable
$z_n^{r[\tau]}$ = binary variable, = 1 if $D_n^{r[\tau]} > 0$, = 0 otherwise, variable
$z^{rr'[\tau]}$ = binary variable, = 1 if $E^{rr'[\tau]} > 0$, = 0 otherwise, variable

Greek letters

$\xi^{r[\tau]}$ = corporate income tax rate paid by currency r, currency/currency, parameter
$\chi^{rr'[\tau]}$ = foreign currency exchange rate from r to r', currency r'/currency r, parameter
$\chi^{r1[\tau]}$ = foreign currency exchange rate from r to 1, numeraire currency/currency r, parameter
$\tilde{\chi}^{r1[\tau]}$ = $\equiv \frac{\chi^{r1[\tau]}}{(1+\rho^{r[\tau]})}$ parameter
$\gamma^{jr[\tau]}$ = opportunity cost of inventory holding paid by currency r, currency/L/year, parameter
$\eta^{r[\tau]}$ = opportunity cost of currency holding paid by currency r, currency/currency/year, parameter
$\mu^{r[\tau]}$ = $\equiv \left(1-\xi^{r[\tau]}\right)\left(1-0.5\eta^{r[\tau]}\nabla t^{[\tau]}\right)$, parameter
$\theta^{jr[\tau]}$ = $\equiv \frac{\left(\mu^{r[\tau]}+\eta^{r[\tau]}\nabla t^{[\tau]}\right)h^{jr[\tau]}+\gamma^{jr[\tau]}}{2} + \mu^{r[\tau]} b^{jr[\tau]}$, parameter
$\pi_{np}^{j'jr[\tau]}$ = customs duty rate of the material moved from storage j' to storage j by transportation process n, paid by currency r, currency/L, parameter
$\pi_i^{r[\tau]}$ = operating cost rate of batch production process i, paid by currency r, currency/L, parameter
$\rho^{r[\tau]}$ = discount rate because of inflation for currency r, currency/currency, parameter
$\omega_m^{j[\tau]}$ = cycle time of customer demand, year, parameter
$\omega_k^{j[\tau]}$ = cycle time of raw material purchasing, year, decision variable
$\omega_i^{[\tau]}$ = cycle time of batch production process i, year, decision variable
$\omega_n^{[\tau]}$ = cycle time of transportation process n, year, decision variable
$\omega^{rr'[\tau]}$ = cycle time of currency transfer from r to r', year, decision variable

$\Psi_i^{r[\tau]}$ = aggregated cost for production process i defined by Eq. 30, currency/L/year, parameter
$\Psi_k^{jr[\tau]}$ = aggregated cost for raw material purchase defined by Eq. 29, currency/L/year, parameter
$\Psi^{rr'[\tau]}$ = aggregated cost for currency transfer from currency storage r to currency storage r' defined by Eq. 32, currency/currency/year, parameter
$\Psi_n^{jj'r[\tau]}$ = aggregated cost for transportation process n to move from storage j to storage j' paid by currency r, defined by Eq. 31, currency/L/year, parameter

Special functions

int[.] = truncation function to make integer
res[.] = positive residual function to be truncated
$|X|$ = number of elements in set X

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Manuscript received Mar. 16, 2010, and revision received Oct. 9, 2010.